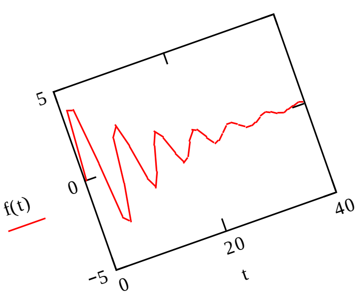
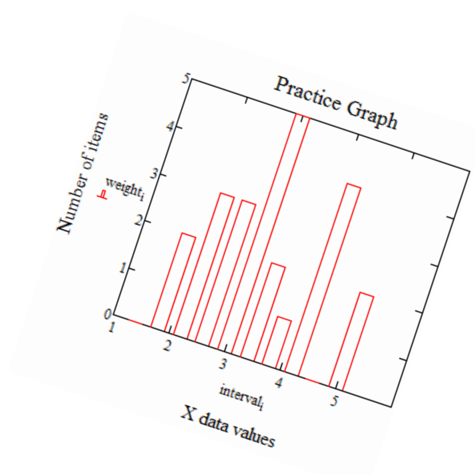
MathCad for Electronics Technology





$$XOe^{\frac{-t}{\tau}} = x \text{ solve}, \tau \rightarrow \frac{20}{\ln\left(\frac{2}{3}\right)}$$

Pearley Cunningham
PRIME 2010

MathCad for Electronics Technology

Introduction

The inspiration for this module is the need for a simple inexpensive text on how to use the MathCad program for solving problems in Engineering Technology. The students at South Campus have studied MathCad in their Technical Computing course for several years and the program is available on the department network. Increased use of the program was dictated by the need to reduce the mathematics load of calculations needed in electronics. This is in response to national trends and recommendations of industry groups. The need for “what if” calculations however remained. Using MathCad the application and use of equations is improved by removing the tedious steps in solutions. It is hoped that with this reference, student confidence in solving the electronics equations will increase.

I would like to thank the students in Technical Computing over the years who have used MathCad in classes and put into prospective how students can use the program. A special thanks to the students in the Northern Tier Basic Electronics Program who inspired me to expand the handouts on MathCad to encompass the full spectrum of math used in the electronics programs. Thanks also must go to Douglas Cunningham, formally of Point Park College, who wrote the first sections on MathCad that was used for years in the Technical Computing class. Some of that original materials has been included in this manual.

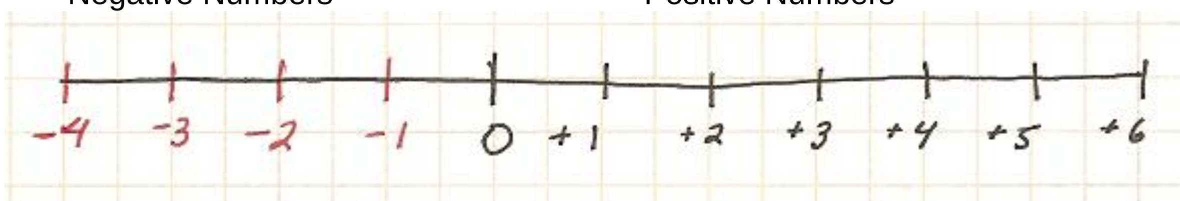
Dr. Pearley L. Cunningham

Chapter 1: Numbers

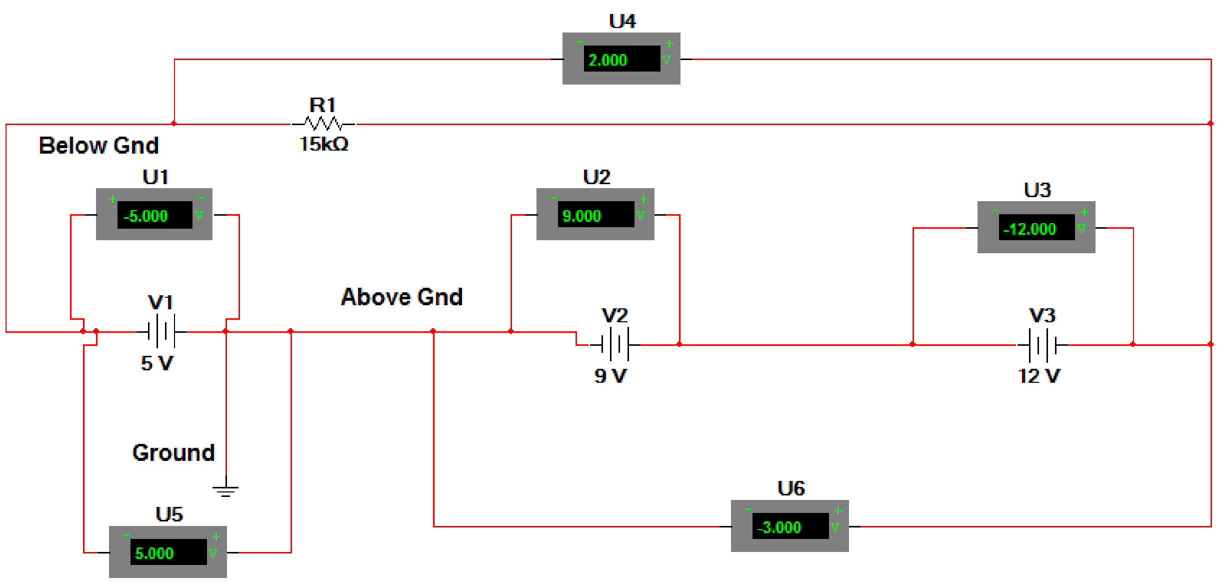
Number Bases and Signed Numbers

The most fundamental concept for math is that of positive and negative numbers. The concept extends from the number line.

Negative Numbers Positive Numbers



Here the positive numbers are those positions to the right of a reverence point (zero). A move to the left is in the negative direction. In electronics we apply this to voltages. A circuit may appear as shown below.



We make the measurements with respect to a fixed reference. This reference is usually referred to as *ground* since many times we use the earth ground as the common reference level. Now if the positive lead of a battery is connected to this ground then the other lead is said to be below ground and is represented as a negative voltage. Voltages are usually taken with the negative (black) lead connected to the ground. As you can see above U1 then reads a negative voltage. U2 reads a positive voltage. Since V3 is connected opposite to V2 the total voltage of the three –end to end- is only 2 volts as seen on U4.
Note that U1 and U5 read the same value but with different sign. This is because they have different reference points. U1 has its reference at ground (the negative lead of meter) and U5 has its reference connected below the ground and hence shows a

positive reading when moving from reference to the (+) meter lead. The meter always reads with the assumption that (-) is connected to the reference point. The total from end to end is the sum of the voltages,

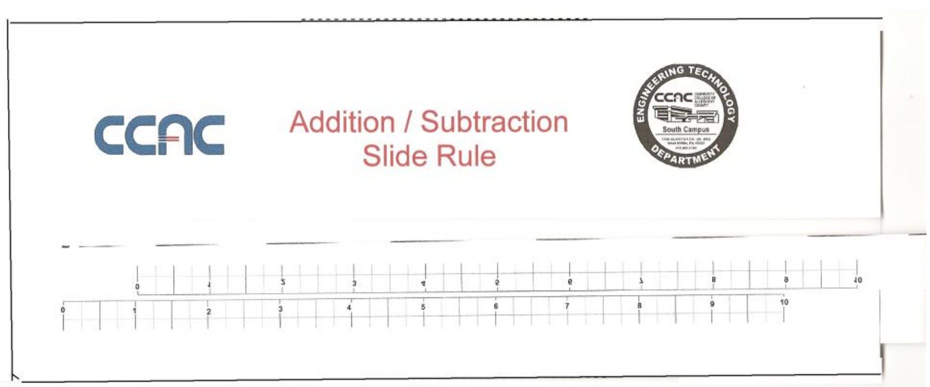
(+5V) + (+9 V) + (-12 V) = 2V
An application of signed numbers.

In electronics this is refered to as Kirchoff's Voltage Law.
Thus by Kirchoff's Voltage Law,
V1+V2+V3 = VT = V4

Adding and Subtracting Signed Numbers

An addition calculator

Prior to the invention of the electronic calculator a device called a slide rule was used. Various versions would do specific operations. Find the file labeled **sliderule1.doc** and **sliderule2.doc** and print the contents or look in the appendix for printed copies of these files. Cut out the two rulers and place into the remaining portion to form an additive slide rule. A picture of the finished device is shown below. This is the first of two slide rules we will design and make in this module.



Solve each of the problems below and then repeat the calculations with the slide rule.

1) +3 + (-2)   2) +4 + (+3)

3) -4 + (-2)   4) +5 – (+3)

5) +5 – (-2)   6) +6 + (-2)

These numbers could be viewed as battery voltages for units wired in series. In this sense we can view negatives as devices reversed from what was expected.

A couple of rules about addition or subtraction of signed numbers are important. First, when adding positive and negative numbers the result is the difference between the two with the sign of the larger magnitude being the sign of the result, as 3+ (-2) = 3-2=+1. Second, if we subtract a negative number from a positive number the effect is to change this to the addition of a positive number as 3- (-2)= 3+2=5.

Also the following situation occurs, -5 – (-2) = -5 +3 = -2.

Another view of negative numbers could be in a monetary application. Suppose we have $50 in a checking account. Then we write a check for $60. The bank honors the check because we have an automatic overdraft feature on our account. Our new balance will be -$10. We have a deficit or debt now of $10. So the negative number represents our debt or the amount of money we need just to be broke!

Multiplication and Subtraction of Signed numbers

We often need to multiply signed numbers. There are specific rules that apply in those cased. These can be summed up by the chart below

Operation	Result
+ x +	+
+ x -	-
- x +	-
- x -	+

When we multiply you do the arithmetic first then apply the sign according th chart above. A common need is for use of Ohm's Law where the voltage is equal to the current times the resistance. This is shown by the formula
V=I *R
We can build a new slide rule to allow the calculation of products like I*R if we base the number line on a log scale. The files Slide3.doc and slide4.doc are such an arrangement, or the appendix has a printed copy of these files.
Fold similar to the additive slide rule. You will need to cut the scale out on slide4.doc to allow it to ride along the edge of the scale on the main page slide3.doc. You will also need to cut along the slide4.doc scale about ¼ inch above the scale to allow the two scales to slide over each other. There are some pictures below showing the arrangement I used. The main scale is labeled I for current, the slider is R for resistance. The Voltage will appear on the I scale as required. See the examples for use.

Note lining up the one and the two give answers for 2x2,2x3,2x4, etc.

If you have an old slide rule you will note that the C & D scales of the slide rule corresponds to our two scales. This will mean you can do all these calculations with a standard slide rule. It is not popular to use the slide rule these days, but it is good to know that you could if necessary. For some operations it is more efficient than the calculator.

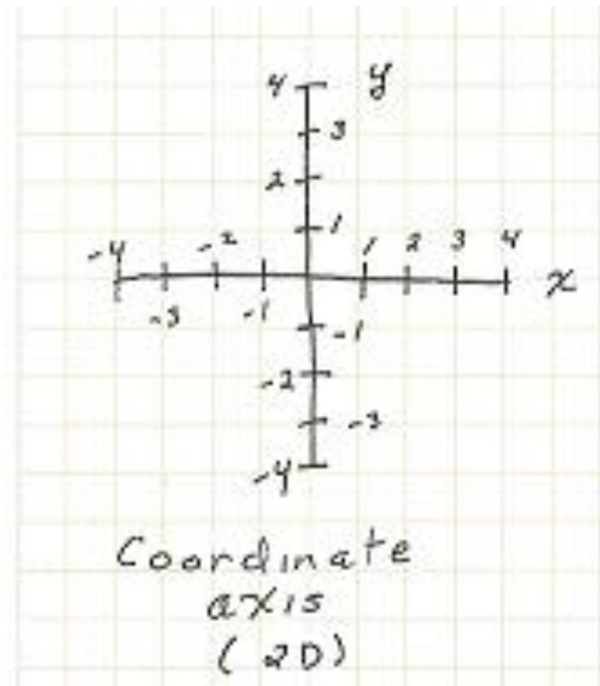
Problems for Practice:

1) 4 x 3   2) 2 x 4   3) 5 x (-2)   4) 6 /3

That last one is the make you think a bit. We know the answer is 2. How will the number lines line up to show this?

Graphs and Number Lines

You may have encountered graphs before in prior courses. The x-y graph is another application of signed numbers as shown in the figure below.



Positive values are represented as to the right and up. Negative values are to the left and down. We will refer to x,y pairs or coordinate pairs. For example, (-2,0) would be the point on the x axis to the left labeled -2. It is understood that the order of the pairs is (x,y). So location (-3, 2) would be left 3 and up 2. Each of these four areas are called quadrants of the x,y plane. In real space we would add a third dimension the z axis coming out at you , but that is hard to draw on a flat page.

Print out the file xypaper1.bmp and plot the points below on the graph paper. There are also some sheets of graph paper in the appendix. As in the picture above put the y on the vertical and the x on the horizontal.

x	y
-3	-7
-1	-3
0	-1
1	1
3	5
5	9

Questions:

Do you see any trend to the data?

Any idea what the value would be if x=6?

Can you draw a line through the points?

Chapter 2: MathCad Operations

The program MathCad is very useful for doing any kind of mathematical operations. It will solve formulas, evaluate equations, do arithmetic, graphs, trig operations and even more advanced topics such as calculus. Skills with MathCad can reduce the burden of calculations for the technician or engineer. An added advantage can be that once a particular problem is solved it can be saved to disk and reused and modified for future situations. It adds one more useful tool to the technicians skill set. We will make a distinction between formulas and equations. A formula is a relationship where we expect a single answer, as in the area of a rectangle. An equation will relate to a situation where we may want multiple values, or need to rearrange the terms to find an unknown quantity. This should become clear as we use them in context.

For a starter let us try something simple like basic arithmetic with signed numbers. For this we will want the Calculator toolbar. This has a good number of items on the toolbar. For now look at the numbers the math keys for +,-, x, and divide.

$2 \cdot 4 = 8$ $\qquad \dfrac{16}{5} = 3.2$

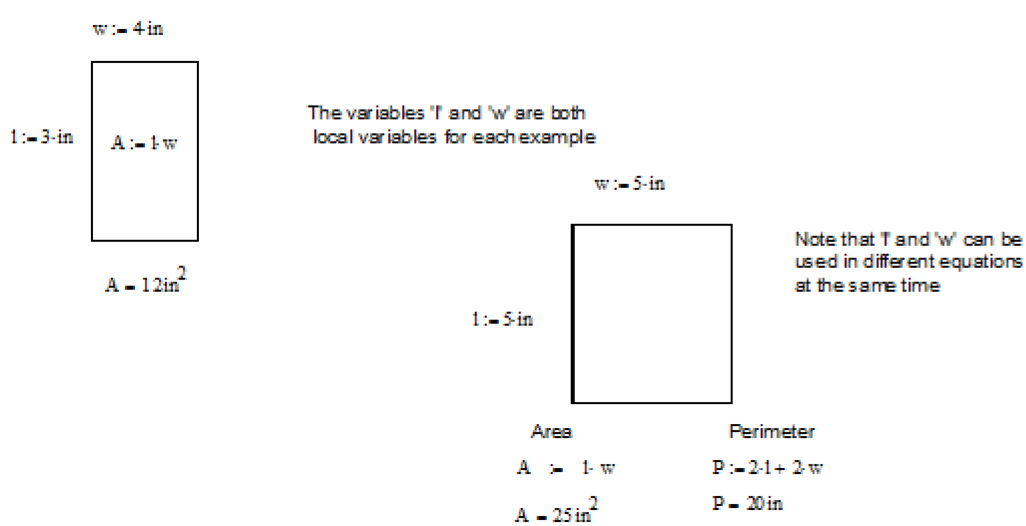
$16 \div 5 = 3.2$

Note how MathCad uses the divide symbol and the / symbol differently although we may think of them as the same.

MathCad is a computer program, so we must be careful to operate within its' defined parameters. Things we may assume, MathCad will not assume. In MathCAD it is important to specify the operation and the type of quantity or variable involved. There are two different ways of defining a variable by defining the variable **locally** or by defining the variable **globally**.

A locally defined variable is a variable that pertains to the current situation at that time. A local variable must be defined before the equation can be written or an error will be indicated.

Example 1: Find the area of these two boxes.



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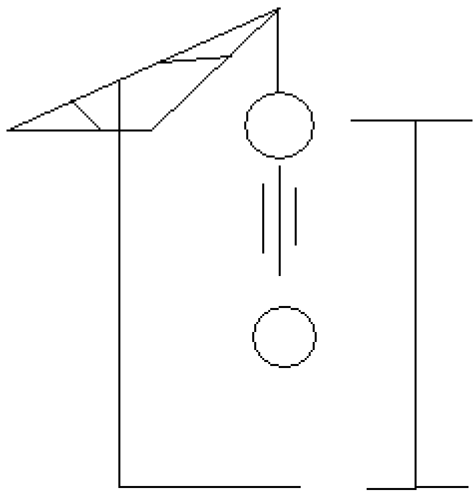
A globally defined variable is a variable that pertains to the document being used at that time. A global variable does not have to be defined before the equation can be written Example 2: The time it takes an object to drop a certain distance.



$$mass := 200 \cdot kg$$

$$H := 10 \cdot m$$

$$t := \sqrt{\frac{2 \cdot H}{g}}$$

$$t = 1.428\, sec$$

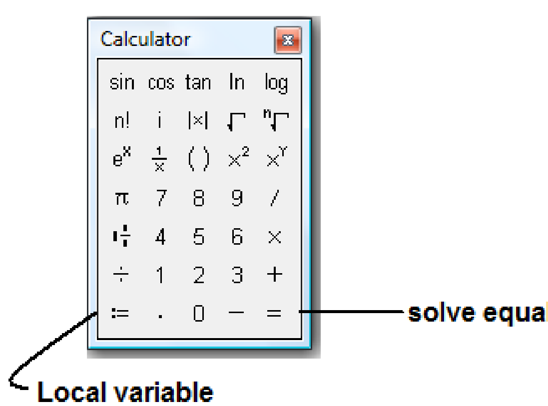
$$g \equiv 9.81 \cdot \frac{m}{sec^2}$$

The variable 'g' is a global variable and the variables 'H' and 'mass' are local variables

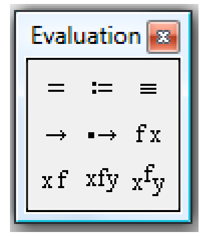
Take note of the three different types of equal signs used in this example. The equal sign that has a colon followed by an equal sign is the symbol for a local variable and a local formula. The equal sign that is made up of three horizontal lines is the symbol for a global variable. The equal sign that looks like an equal sign is really an equal sign and is used to determine the solution, that is evaluate the result. The numerical value of ‘t’ was determined in this manner. MathCad is a top – down program. Notice since ‘g’ is a global variable it may appear before or after ‘t’ on the page. MathCAD seeks through the entire document for the global values. The local variables must always come before the formula on the page in which they are used. Local variables that come after the equality will not affect the results that come before them on the page.

There are several ways to type these symbols. They are (a) use the keyboard, and (b) use the mouse and tool pallets.

The tool pallets are shown here.



Using the keyboard
To get the symbol for an equal sign type **shift**+**;**
To get the symbol for a global equal sign type **shift+ ~**
To get an equal sign type =



Using the mouse and tool palette
To get the symbol for any of the equal signs go to Calculator Toolbar or the Evaluate Toolbar. Choose the correct symbol off the list. It is a good idea to always keep these toolbars on the desktop. The global equal is on the evaluate toolbar shown here. The other two equal signs are also on this toolbar.

How to set up a formula
The way to set up a formula is as follows:

1) Set up variables either local or global
2) Define the formula in terms of the variables that were set

For example the area of a rectangle is A= L*W
We can use full names also for variables as Area= Length * Width. This is sometimes helpful to keep track of what each item represents.

$\text{Length} := 5 \qquad\qquad \text{Width} := 12$

$\text{Area} := \text{Length} \cdot \text{Width}$

$\text{Area} = 60$

If we define Width after the Area definition we get errors.

$\text{Length} := 5$

$\text{Area} := \text{Length} \cdot \text{Width}$

$\text{Area} = \blacksquare$

$\text{Width} := 12$

MathCad does not know what Width is since we did not define it before it was needed. Now if Width is a global variable, this would not matter as below.

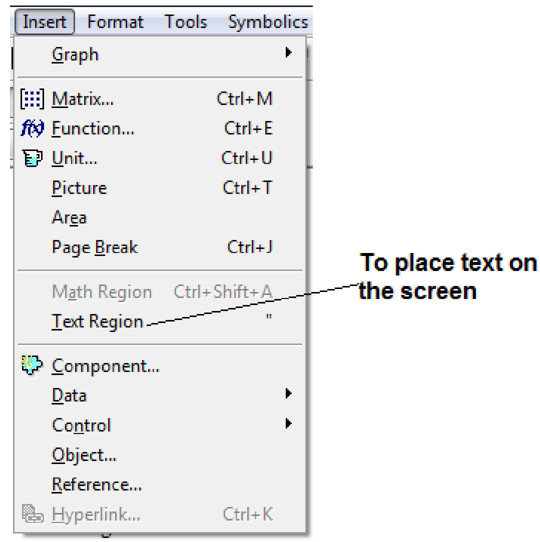
$\text{Length} := 5$

$\text{Area} := \text{Length} \cdot \text{Width}$

$\text{Area} = 60$

$\text{Width} \equiv 12$

It is important to be clear about the variable! One final note, in solving a problem you may wish to place text on the screen. For example, label the top of the page with “Problem 1”. To do this, place the cross-hair at the place you wish the text to begin: select “Insert” at the top of the screen, then click on “Text Region”. Begin typing text at the red vertical cursor. Delete will erase typed items and clicking on a blank space below or above the text will end that text entry and return you to equation entry.



Application: Sample exercises

1) Use MathCAD to find the area of a rectangle with a length of 10 and a width of 5.

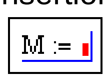
2) Use MathCAD to find the area of a circle with a diameter of

3) The formula for Ohm’s Law is Volt = AMP x Resistance. Use these variable names and find the volts for a current of 3.4 Amps and resistance of 28 Ohms.

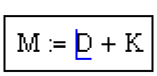
The ART of editing

Editing in MathCAD is easy. MathCAD editing falls into two categories: 1) formula editing or 2) text editing. Text editing is the easier of the two. To edit text, click outside the text area, move the mouse to the text area that is to be edited and click the left mouse button at the point you wish to edit, then begin either typing new text, deleting, or modifying.

Formula editing is a little harder, but this is also where the ART comes in. There are two types of cursors employed to edit. The crosshair (in red) is used to locate a new formula, text region, plot region, or variable assignment. The insertion point is a blue L that allows the insertion or deletion of any single character. The placeholder box and insertion point are shown below:



The blue L that is around the placeholder indicates where in the formula new variable information can be placed. Here the new material will go on the left of the L.



The blue L indicates where in the formula editing can be done. Here the new material will go on the right.

The insertion point is produced by clicking on a variable, function, or a number. To move around inside an equation or any other MathCAD formula object all that has to be done is use the arrow keys. The up arrow moves the selection point upward in the equation. The down arrow moves the selection point downward. The left arrow moves it to the left and the right arrow moves it to the right. The spacebar can be used to expand the area covered by the blue L.

The equation below needs a square root inserted for the square root to go in the correct place the blue insertion point needs to encompass everything that needs to be in that square root. When the square root button is then pressed it will insert it over everything that was marked.

$b := 2$ $\quad a := 3$ $\quad c := 7$

$\boxed{x := b^2 - 4 \cdot a \cdot c}$ $\qquad \boxed{x := \sqrt{b^2 - 4 \cdot a \cdot c}}$

To place a minus sign in front of the square root you must use the arrows to position the blue L with the vertical line on the left and covering the whole equation. Then all that needs to be done is enter a minus sign.

$\boxed{x := \sqrt{b^2 - 4 \cdot a \cdot c}}$ $\quad \boxed{x := -\sqrt{b^2 - 4 \cdot a \cdot c}}$

Rearranging the document

Your MathCAD document is no different than any engineering problem solving document. Information should flow down the page. This enables the reader to easily follow your line of reasoning and understand what you are trying to communicate. Suppose you did not leave sufficient room in your document to insert text or a drawing above your calculations? To move any expression or text region in MathCAD, just left click above and to the left of the expression or area that needs to be moved. Hold down the left mouse button and drag a dotted box over the items to be moved. Each region to be moved will be boxed separately. Release the mouse button. Next, left click the mouse inside any one of the boxes and hold it. You can now drag the entire set to a new position. WARNING: remember MathCAD’s local variables are page position sensitive. If you move an expression above the definition of its variables, you will get an error. If this happens, just move it lower on the page to redefine the local variables.

Chapter 3: Equations

Linear Equations

Suppose we have the following situation:

$$3 + ? = 7$$

And we ask the question what is the missing value. Most of us could figure we need the number 4. Now If instead of a question mark, we use a letter, say x, we would have

$$3 + x = 7$$

This would be a typical expression from the language called Algebra.

I used the term language because it can be viewed as a sentence just like in English. This language describes how quantities relate to each other. These sentences are called equations.

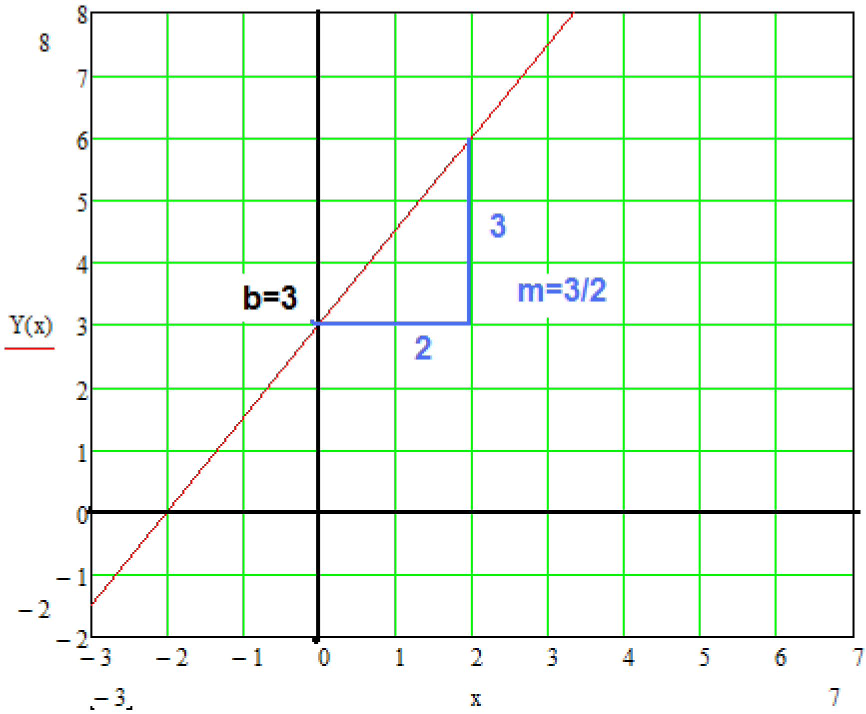
In engineering we need to be able to make predictions about what will happen. To do this we need to quantify the situation. This leads to the use of equations. The equation will describe in a quantitative manner an electronic circuit or device. This allows prediction of how the device will behave in future applications.

A straight line on a graph can result in an equation of the form

$$y = m*x + b$$

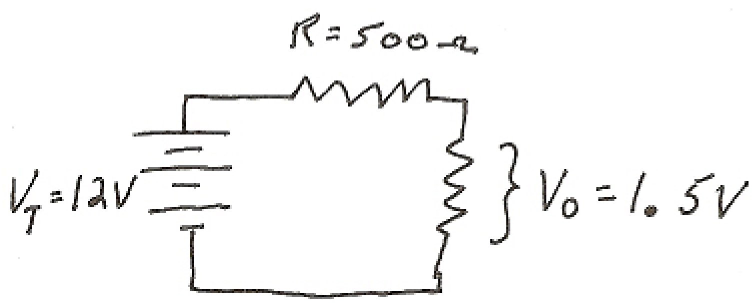
This is called a linear equation since it results in a straight line on a graph. The x and y represent the ordered locations along the line. There are an infinite number of possible values on the line. The m term relates to the angle or slope that the line takes in moving up on the graph. The b term is the location on the vertical axis that the line crosses. If we have the line drawn on the graph, we can read this value off the graph directly. The m or slope term is a bit more difficult to obtain. The technique is to draw a right triangle on the graph and determine (to the scale of the graph) the lengths of the sides. By dividing the vertical by the horizontal distance we can determine the “rate” of climb. For example, if the ratio is 3/2, for every 2 units on the x we go up 3. The slope would be listed as 1.5, since in engineering we prefer decimal notation when possible.

The sketch below will show you just such a line on a graph. This was drawn using MathCad and y= 1.5 X+ 3.



Y=1.5x + 3

Many times we will have only one unknown quantity. In Algebra this is often represented with the letter x, y or z. However, in electronics the letters V, I and R are more common. This makes no difference to the math. So don't get hung up on the letters used to represent the variable or unknown quantity. From the circuit below a typical equation from Kirchoff's Law would be



$V_T = I*R + V_0$

$12 = 500\,(I) + 1.5$

In this equation we do not know the value of the current I and it is represented in the equation as a letter. We would like to solve for this quantity.

To do this there are a few "rules" we must always follow when dealing with equations.

1.What ever is done to one side of an equation must be done on the other side. Add, Subtract, Multiply and Divide. Are the most common operations.
2. Do the above until only the variable letter is left on the left side of the equation.

3. Observe “order of operations” with respect to parentheses in the equations. Remember multiply and divide comes before add and subtract.

Below is a step by step application of this to the equation above.

$$12 = 500I + 1.5$$

Subtract 1.5 from both sides

$$12 - 1.5 = 500I + 1.5 - 1.5$$

$$10.5 = 500I$$

Divide each side by 500

$$\frac{10.5}{500} = \frac{500I}{500}$$

$$.021A = I$$

flip sides

$$I = 0.021A$$

Often it is helpful to perform a “check” of the solution. This can be done by putting the value discovered into the equation and seeing that the left and right side are the same.

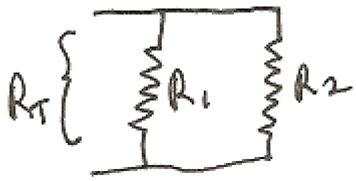
$$12 = 500I + 1.5$$

$$12 = 500(0.021) + 1.5$$

$$12 = 10.5 + 1.5$$

$$12 = 12$$

In addition to equations we often have formulas with which to deal. There is little difference between them, mainly in how we intend to use them. An equation can have the variable anywhere, but a formula has the unknown on the left side and other letters representing quantities on the right. We typically know the value of all those quantities. The formula for a parallel pair of resistors is below. The values of R1 and R2 are known.



$$R1 = 500\,\Omega$$
$$R2 = 750\,\Omega$$

$$R_T = \frac{R1 \cdot R2}{R1 + R2}$$

$$RT = \frac{500 \cdot 750}{500 + 750} = \frac{375\,000}{1250} = 300\,\Omega$$

However, sometimes the typically known variable is what we want, then it becomes an equation. One thing to get use to in engineering, and electronics especially, is that we will use equations and formulas that we are not fully knowledgeable about. That is we do not have to be able to derive the equation ourselves, if we trust the source of the equation –text book, expert, device manufacturer.

Solving Equations

How do we solve equations for a single unknown. The section below is from the help menu of MathCAD.

1. In your worksheet window, type the equation using the Boolean equal sign. To do so, type **[Ctrl] [=]** or click the Equal To button on the **Boolean** toolbar.

$$\boxed{x^2 - 4 = 21}$$

2. Click anywhere in the equation and press **[Ctrl] [Shift] [.]** to insert a placeholder followed by the symbolic equal sign.

$$x^2 - 4 = 21 \;\blacksquare \rightarrow$$

3. Type the keyword "solve" in the placeholder and press **[Enter]** to see the result displayed.

$$x^2 - 4 = 21 \text{ solve} \rightarrow \begin{pmatrix} 5 \\ -5 \end{pmatrix}$$

Note: If the equation contains more than one variable, you must also type a comma after "solve" and then type the variable you want to solve for in the placeholder that appears.

The following is an example of solving for a variable. First input the equation, the steps will look as below:

$\boxed{3 + x}$ $\boxed{3 + x = \blacksquare}$ $\boxed{3 + x = 7}$

We have used here a special form of the equal sign. It is called a Boolean Equal. You produce it by Pressing the Control Key **at the same time** as the equal key – step one above.

Then place the curser anywhere in the equation and press the keys Control – shift-period all at once (see step two above.) When the enter key is pressed the answer will appear at the right of the equation as shown.

$\boxed{3 + x = 7 \ \blacksquare \rightarrow}$ $\boxed{3 + x = 7 \text{ solve} \rightarrow}$ $3 + x = 7 \text{ solve} \rightarrow 4$

Use this technique to solve all of the sample equations presented.
Sample equations:

1) 2*x + 17 = 4

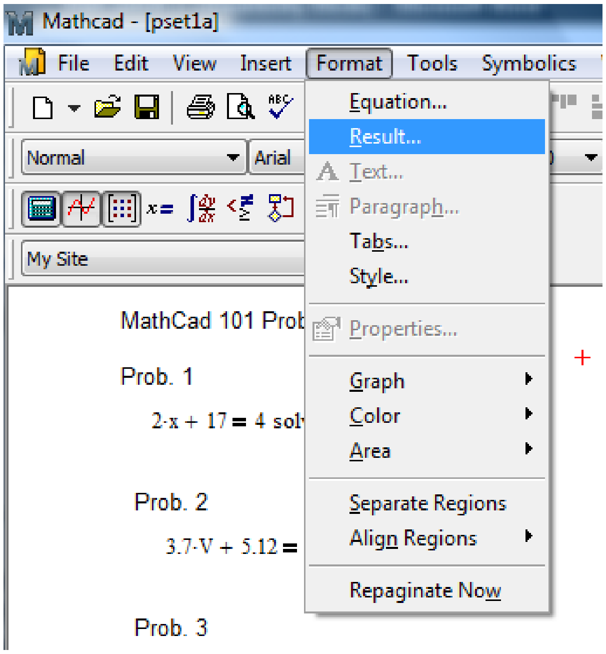
2) 3.7 * V + 5.12 = 12.5

3) 7 * y – 14 = 3.9

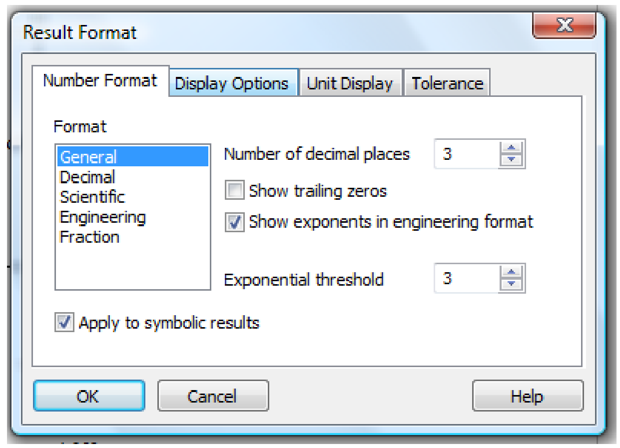
4) 16 -3 * R = 9

5) 2.7 *A + 12 = 17

I used “ugly” numbers above because in engineering we seldom have integers in our equations. In a Math class the examples usually have integer numbers and the answers will be either whole numbers or common fractions. The solutions for these are in file **pset1.mcd .**



In the file the answers are given to way to many decimal points. To force the display to something reasonable for electronics, say 3 decimal points, click on Format> Results and then set the screen as shown. Be sure to check off the box as shown.



File **pset1a.mcd** shows the result of this setting.

Application:

To demonstrate that we understand the processes above, solve the five equations below using only paper and pencil. Fill in your answers to the data box provided. Then – only after the hand effort- use MathCad to find the solutions and enter those values into the data box.

1) 3*R +2=12

2) 14=6.5 *A -7

3) 9 -2.7*Q =3

4) 18*x +5 = 12

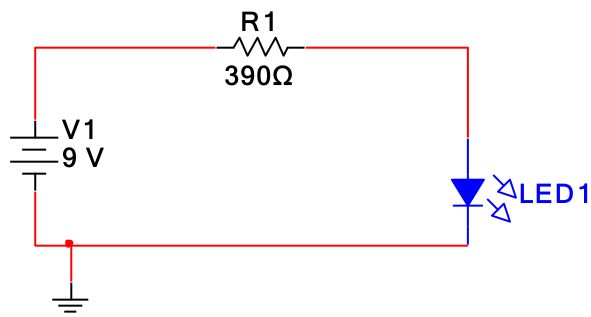
5) 5.6 + 3*L = 17

Data Results:
Enter the unknown value in each square below.

Equation #	Hand Method	MathCad Method
1 (R=)		
2 (V=)		
3 (Q=)		
4 (x=)		
5 (L=)		

Another trick is that the equation can substitute values as it solves the equation.
For example:

This is the general circuit for the LED.
Below is the solution form MathCad for the 9V shown and for two other values.



$R := 390 \qquad V := 9 \qquad VO := 1.5$

$V = I \cdot R + VO \text{ solve}, I \rightarrow 0.019$ Amperes

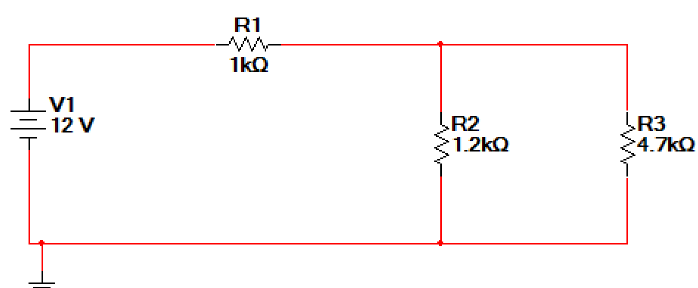
$R := 390 \qquad V := 12 \qquad VO := 1.5$

$V = I \cdot R + VO \text{ solve}, I \rightarrow 0.027$ Amperes

$R := 390 \qquad V := 18 \qquad VO := 1.5$

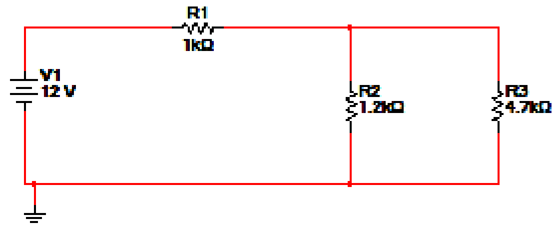
$V = I \cdot R + VO \text{ solve}, I \rightarrow 0.042$ Amperes

The Mathcad file LEDcirc.mcd will provide the above solution. Note that the solve has had the variable name listed for which we wish to solve. This will work for any equation with multiple values involved. A typical circuit in Basic Electronics is be able to solve the following circuit :



Suppose we wish to solve for total current. We need the total resistance and then can solve for the current. In MathCad this will look like the following.

Intro Target Circuit



$R1 := 1000 \qquad R2 := 1200 \qquad R3 := 4700$

$V1 := 12$

$$RT := R1 + \frac{[R2 \cdot (R3)]}{R2 + R3}$$

$$V1 = IT \cdot RT \text{ solve}, IT \rightarrow \frac{177}{28850}$$

This is file **serpar circuit.msn** on the book CD. The fraction at the end is easily converted to a decimal of 0.0061 Amperes. The solve feature finds an exact solution if possible, and a decimal would be only approximate. Notice that with the MathCad solution the steps are in agreement with the Process described in the paper on **Solving Problems Like A Pro** found in the appendix.

Quadratic Equations

A quadratic equation is one that has one term that is squared. The typical form for this equation is

$$Y = ax^2 + bx + c$$

This type relationship is handled in the same manner as the linear equation or formula. Many times this format will come up with either area calculations, power relations, or some kinematics equations in Physics.

For example, the distance a object falls is given by the formula

$$X = 1/2\ (a)\ t^2$$

Where a is the acceleration of the object. If falling this is 32 ft/sec^2.

In MathCad this looks like

$a := 32 \qquad t := 3$

$$x := \left(\frac{1}{2}\right) \cdot a \cdot t^2$$

$$x = 144$$

Another situation is shown below:
Example 1: $y=a(1-x)^2+b$

$a := 2 \qquad b := 3 \qquad x := 5$

$$y := a \cdot (1 - x)^2 + b$$

$$y = 35$$

Advanced Equation Techniques

Now let's say that you want the values for y related to the x values between -2 and +2. MathCAD considers this type of operation a mathematical function. A function does not imply a single answer, but a relation between the variables.

Example 2: $y(x)=a(1-x)^2+b$

$a := 2 \qquad b := 3 \qquad x := -2, -1 .. 2$

'x' is the independent variable
and has a range from -2 to +2.

$$y(x) := a \cdot (1 - x)^2 + b$$

The function y(x) is dependent on the values of 'x'

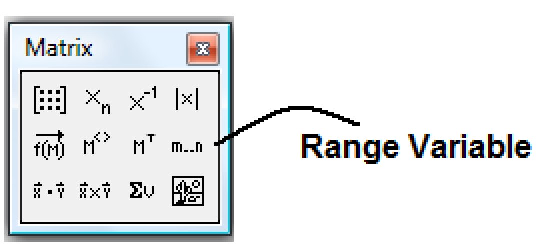
$y(x) =$

21
11
5
3
5

The answer is the set of values calculated from the 'x' values. The number of calculated values is based on the step of the range values. In this example the step is one.

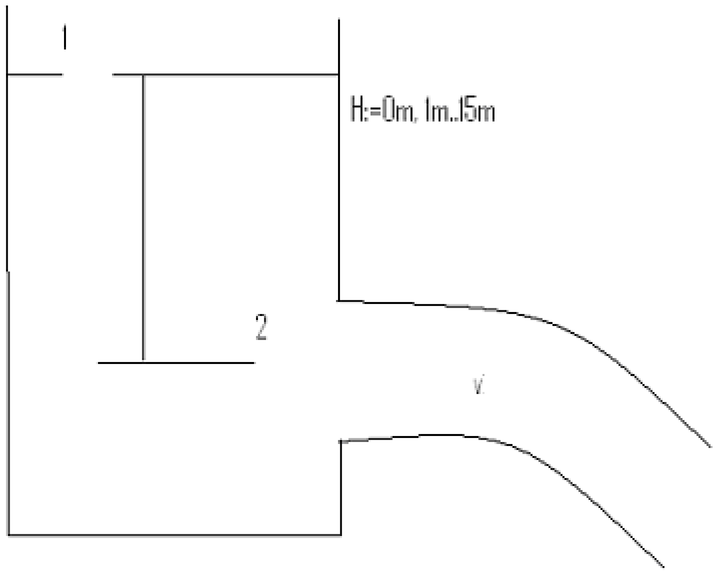
The range for Example 2 is defined as -2, -1, 0, 1, 2. The way MathCAD determines these numbers is by being given the first number in the range, the next value (this determines the step), and the last value in the range.

Using the tool pallet type the variable, the appropriate equal and the first value then click m..n on the pallet. And follow below.



For the keyboard, type in the range as follows:
x Shift+; -2 , -1 ; 2
Type the function for the equation as follows: f (x) Shift+; a (1 – x) Shift+6 2 Spacebar Shift+= b
To obtain the answer type: f(x) =

An example of using a functional equation would be Torricelli’s Theorem as shown in Robert L. Mott’s book Applied Fluid Mechanics Third Edition.



$$g \equiv 9.81 \cdot \frac{m}{sec^2}$$

Torricelli’s Theorem

$$v(H) := \sqrt{2 \cdot g \cdot H}$$

Solution for H

For this example, the calculation of Torricelli's Theorem yields the velocities of a fluid for varying fluid depths.

This information could then be used to determine the discharge rate for a given opening.

$H =$ (m)

0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

$v(H) =$

So we have three types of relationships,

1. formula

    This is a relation where the unknown is on the left and all values on the right are known.

2. equation
    A relation similar to the formula, but in which you may wish solve for something other than the left hand value.
3. function
    An equation where no single value is wanted but a range of values. Y(X) = pronounced as Y of X equals …

Polynomials

A polynomial is an equation that has a power relation with in it. These can be either whole or fractional powers. For example

$Y := a^3$ $\quad\quad$ $Z := \sqrt{x^4 + 35}$ $\quad\quad$ $V := 3x^3 + 2 \cdot x^2 - x + 3$

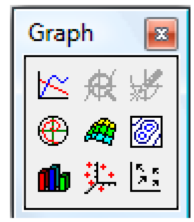
MathCad will treat any of these relationships with equal ease. If you can type it, MathCad can solve it. The equations will look just like they appear in the textbook, which makes it easier to compare your version to the text version.

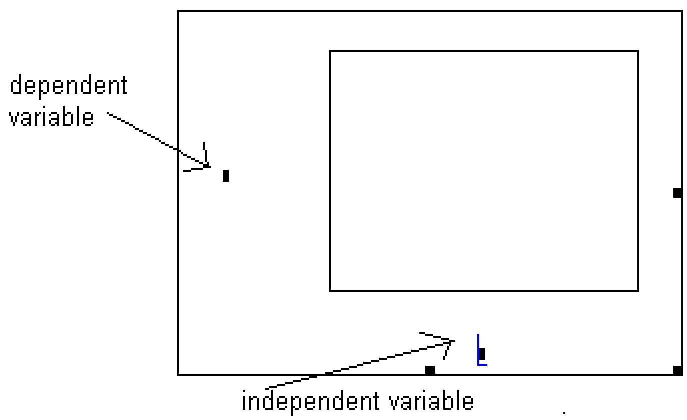
Chapter 4: Basic Graphics

Graphing Results

In Engineering Technology and Science it is often necessary to produce a graph of the functional behavior of an equation. In the last chapter, we established data tables that could be used to graph the function. In this chapter MathCAD will bypass the data table and produce the actual graph…after all, one picture is worth a thousand words.

Graphs usually involve two different variables. The horizontal is the independent variable and the vertical is the dependent variable. The independent variable will be defined as a local range variable. For example **t:=1,2..10**. The dependent variable will be a local function of t. For example **y(t):=3*t^2 + 2**. To produce the graph of the expression, first place the crosshair at the location for the upper left corner of the graph and click once. Then select the Graph toolbar at the top of the screen. From this toolbar, select X-Y Plot. At this, a box will appear at the location of the crosshair.

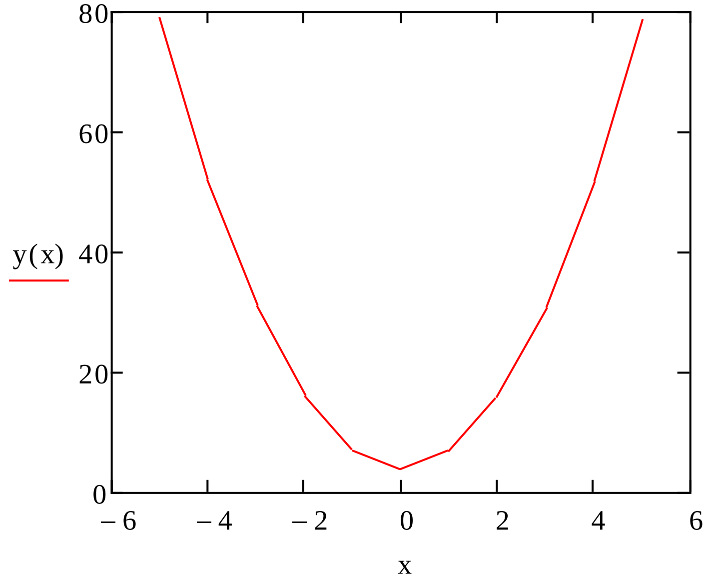




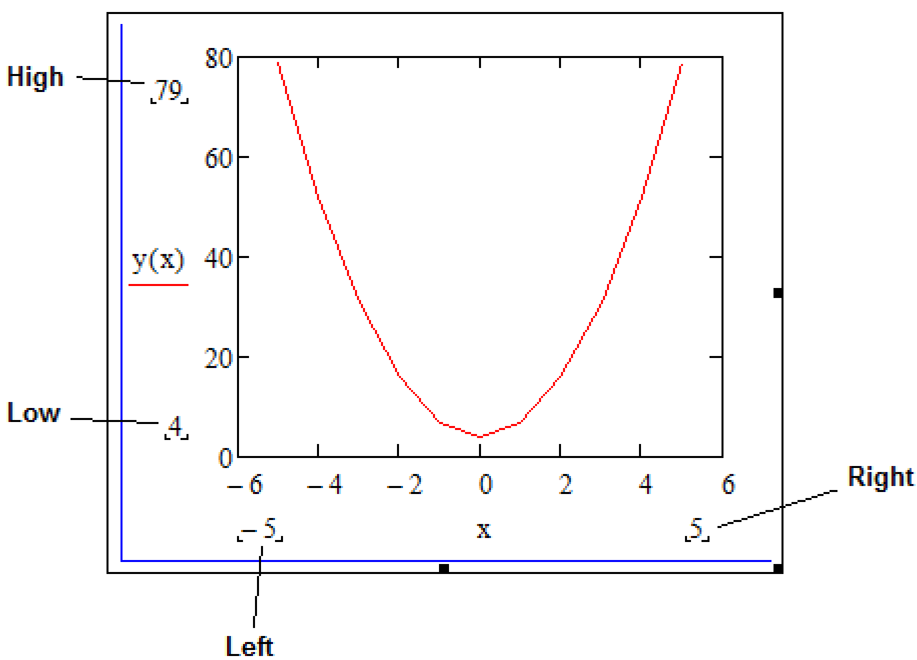
The blue edit L will be over the independent variable. Type the local range variable here, then press the left cursor 4 times to move to the dependent variable in the middle of the vertical axis. Type the function variable here exactly as it appears to the left of the := sign, remember it is a function. The remaining marked locations are for changing the limits of the graph. Use the left or right arrow keys to position the marker over these areas and type the desired values or skip these to accept the MathCAD default decision. In the example below we have a quadratic equation with the resulting parabola shape.

$$x := -5, -4 .. 5$$

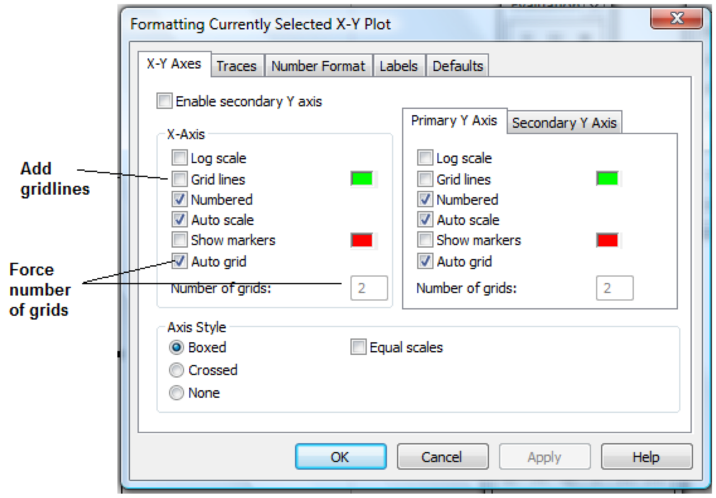
$$y(x) := 3 \cdot x^2 + 4$$



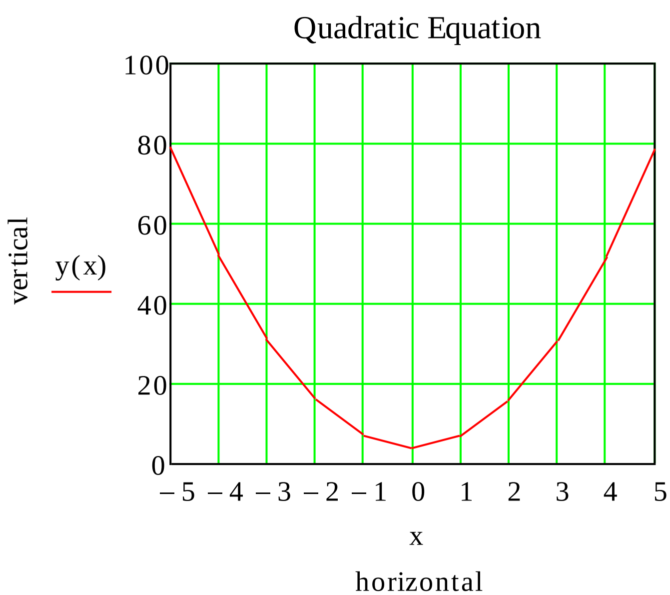
This graph is using the MathCad default values. We can change that by overriding the graph values. Click on the graph to place a black box around the graph as below and change the limits as needed.



In addition to changing the range you may wish to do other things like change the color of the display, add grid lines to the graph, or title the graph.



The graph has been modified using this toolbar to the style below.

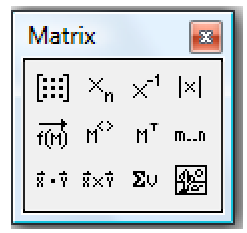


This graph can be copied and pasted into other documents for lab reports and test reports. This will give your work a professional, finished appearance.

Graphing Data Points

There are many times when we want to draw a graph of data points collected in the lab or shop. This maybe for analysis reasons or just to have a presentable display. In any case MathCad has some ability in this area. To input data one must use the Matrix Toolbar. A matrix is an ordered array of numbers. In the case of lab data it will usually result in a single column of numbers with multiple rows. Suppose we have 10 data points and wish to plot them. We will need the Matrix toolbar as shown on the below. Set a local variable as

X := then select the  icon to produce the display below. This sets up a matrix. Indicate 1 column and a number of rows to match the amount of data you have.

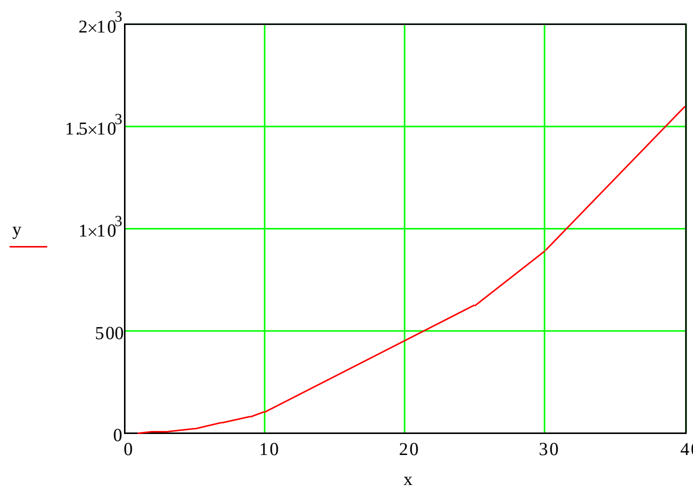


Then type each of the necessary numbers into the space holder squares. MathCad will keep track of the number of entries made and pair them to any additional variables plotted.

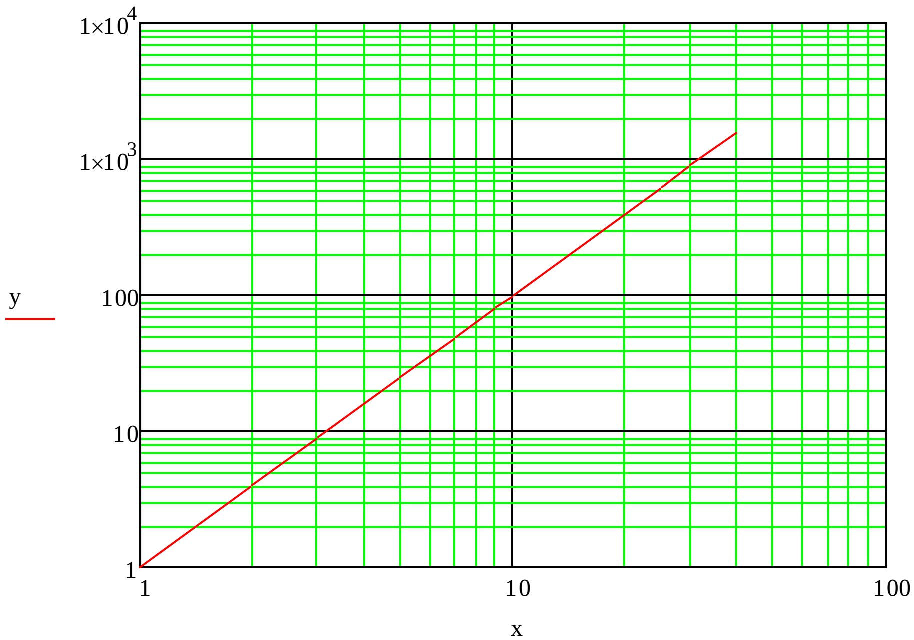
$$X := \begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix}$$

The following shows how this is done and then a graph is drawn in the same manner as with a function. We don't have to indicate a *function of* feature because we have discrete values and equal numbers of both.

$$x := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 5 \\ 7 \\ 9 \\ 10 \\ 25 \\ 30 \\ 40 \end{pmatrix} \qquad y := \begin{pmatrix} 1 \\ 4 \\ 9 \\ 25 \\ 49 \\ 81 \\ 100 \\ 625 \\ 900 \\ 1600 \end{pmatrix}$$



This is a quadratic function. We could quickly confirm this by plotting on log-log paper. The result will be as shown below, and the straight line will confirm a power relationship. (See the appendix on graph shapes.)



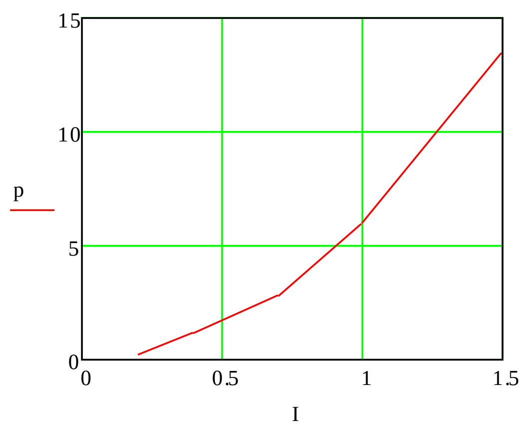
Quadratic equation plotted on log-log paper resulting in a straight line.

It is also possible to calculate new values once the data is entered. For example you may be more interested plotting the power versus current in a current –voltage experiment. As shown below,

$$v := \begin{pmatrix} 1 \\ 3 \\ 4 \\ 6 \\ 9 \end{pmatrix} \qquad I := \begin{pmatrix} .2 \\ .4 \\ .7 \\ 1.0 \\ 1.5 \end{pmatrix}$$

$$p := \overrightarrow{(I \cdot v)}$$

$$p = \begin{pmatrix} 0.2 \\ 1.2 \\ 2.8 \\ 6 \\ 13.5 \end{pmatrix}$$



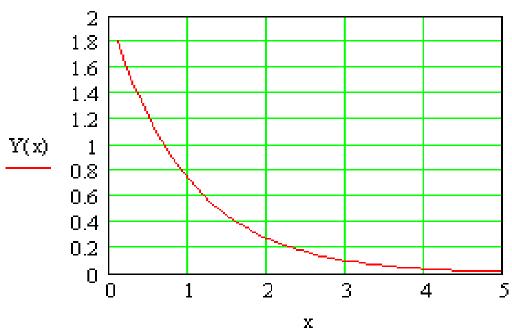
To accomplish this operation we need to use a special operation called “vectorize” which will allow item by item multiplication.
 First set up the I v term place the I v under the blue edit bar, then select the vectorize icon 

Use the = sign to see the resulting values .

Chapter 5: The Number e

The number e is one of a group of numbers special in mathematics. It is an irrational number, which means it can not be represented as a common fraction and is a never ending non repeating decimal. The number π is another irrational number. But as we use 3.12 for π, we could use 2.72 for e as a similar approximation. So when we see e^x in a formula, it is like having 2.72^x in the formula. However, both the value of π and of e are built into MathCad to the accuracy of the program.

This process of cooling was first recorded by Issac Newton (of the falling apple fame) and is called Newton's Law of Cooling. This relationship explains the way in which the temperature of a cup of coffee will cool over time. Of course, it also applies to a cooling block of steel, or the temperature inside you house when the furnace goes off in the winter. It takes the format of $x = X0\ (e^{-t/\tau})$ where t is time and τ (tau) is called the time constant. $X0 = X_{high} - X_{low}$ and x is the amount above the low reading. In general the curve will appear as shown below.



One prominent feature of the e^x curves is that in a fixed unit of time (the time constant, the Greek letter tau, τ) the curve will drop by 63% in one τ and in 5τ it will drop to nearly level.
For your coffee this means it is at room temperature. Knowing the time constant of a system can be useful. This is a constant of the system and does not depend on the high or low points. So, if we know the time constant for a house, we can predict the temperature after the power has been off for 2 hours, 4 hours, etc. We can predict when the temperature will be low enough to freeze the pipes. For a block of steel it allows us to know when it will reach a reasonable temperature to handle.

Example:
Coffee is sitting at 180°F and room temperature is 72°F..(a) if the time constant of the coffee cup is 120 seconds, what will be the temperature after 5 minutes? (b) When will the coffee be at room temperature?
p.s. McDondald's serves it's coffee at 180°F.

First find the temperature difference,
$X0 - T_{high} - T_{low} = 180 - 72 = 108\ ^oF$
5 minutes = 60 x 5 = 300 seconds
With the time constant at 120 seconds we get
$X = X0\ e^{-t/\tau} = 108\ e^{-300/120}$
By calculator this is x=8.9
Or we could use MathCAD and get…

$$X0 := Thigh - Tlow$$

$$Thigh := 180$$

$$Tlow := 72$$

$$t := 300$$

$$\tau := 120$$

$$x := X0 \cdot e^{\frac{-t}{\tau}}$$

$$x = 8.865$$

Temperature change left after the 5 minutes

Current temperature will be

$$72 + x = 80.865$$ Degree F

In 5 time constants or 10 minutes it will be at room temperature

What could be done to extend the time the coffee will stay warm?
Would the type of cup effect the problem in any way?

Example:
Finding the time constant of a house. A house is sitting at 72 ^{o}F with an outside temperature of 30^{o}F. The power goes off for 1 hour and the temperature goes down to 65 ^{o}F. What is the time constant of the house? Let's apply MathCAD to this problem.

$$Thigh := 72 \qquad Tlow := 30$$

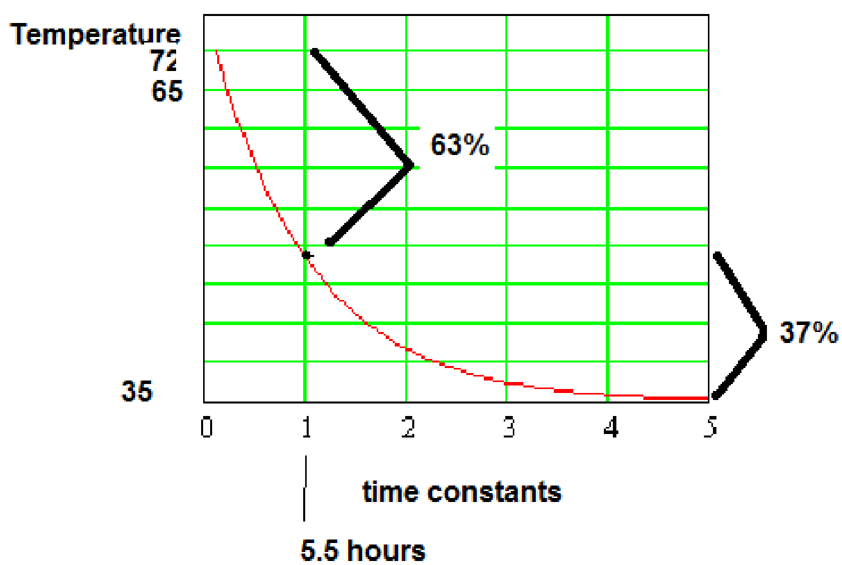
$$t := 1 \text{ hour} \qquad x := 65 - Tlow$$

$$XO := Thigh - Tlow$$

$$XO \cdot e^{\frac{-t}{\tau}} = x \text{ solve}, \tau \rightarrow -\frac{1}{\ln\left(\frac{5}{6}\right)}$$

$$-\frac{1}{\ln\left(\frac{5}{6}\right)} = 5.485 \quad \text{hours}$$

note that the ln value is negative so the time comes out positive.



Example:
A manufacturer is making a new Thermos Bottle, and they want to keep the coffee at a temperature of at least 120°F for 40 minutes. If we assume the coffee is starting at 180 °F, what time constant will the thermos need to have?

$TempHigh := 180$ $\qquad$ $TempLow := 72$

$TempTarget := 120$ $\qquad$ $XO := TempHigh - TempLow$

$x := TempTarget - TempLow$

$t := 40$

$$XO e^{\frac{-t}{\tau}} = x \text{ solve}, \tau \rightarrow -\frac{20}{\ln\left(\frac{2}{3}\right)}$$

$$-\frac{20}{\ln\left(\frac{2}{3}\right)} = 49.326 \qquad \text{Minutes}$$

So the time constant, τ, needs to be 49 minutes. In 5τ or 245 minutes or 4.1 hours the contents will be at room temperature, 72 °F. I would suggest they set their sights a little low. If filled on the way to work, the coffee won't make it to noon.

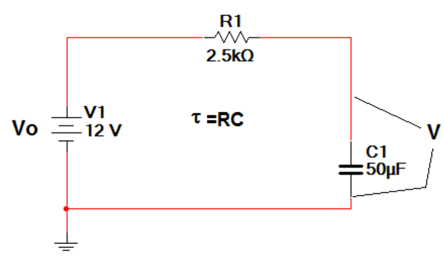
Charging a Capacitor

In electronics a common event is to charge and discharge a capacitor through a resistance. If the capacitance is in μF and the resistance is in KΩ then the product of these would be in milliseconds (ms) and is the time constant of the circuit, or τ=RC
For a discharge circuit the equation is

$V=Voe^{-t/\tau}$ We have considered this equation in the prior section, and it is no different when talking about voltage than when involved with temperature.

For the charging of the capacitor the equation changes to …

$V=Vo(1- e^{-t/\tau})$ the circuit is shown at the right.



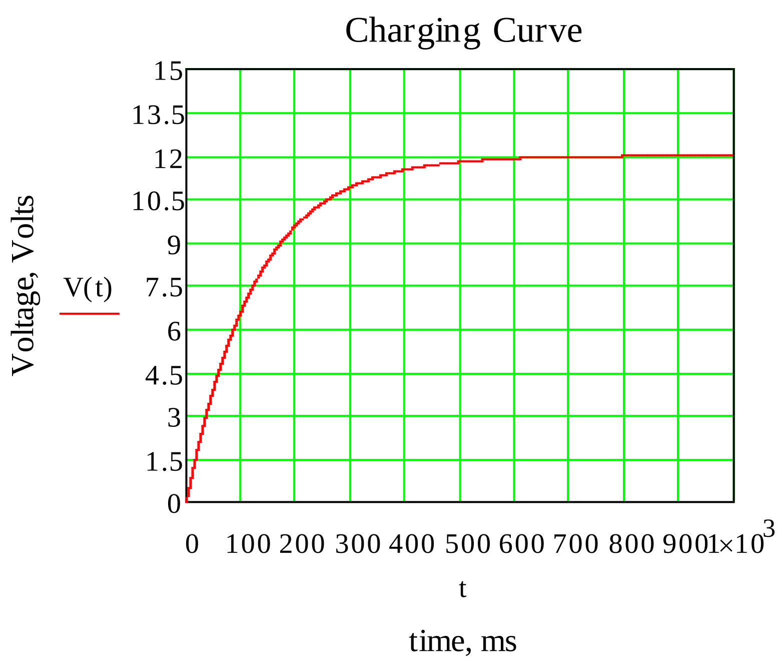
Initially the voltage on C1 will be zero because,

$e^{-t/\tau} = e^{-0}$ since t=0 and anything to the zero power is equal to one. Of course t=R1 C1 or 125 ms. MathCad will treat this as shown below.

$Vo := 12$ $\qquad R1 := 2.5$ $\qquad C1 := 50$

$\tau := R1 \cdot C1$ $\qquad t := 0, 0.1 .. 1000$

$$V(t) := Vo \cdot \left(1 - e^{\frac{-t}{\tau}}\right)$$

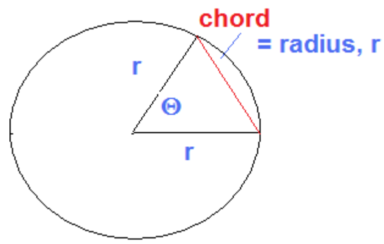


Here again in one time constant 63% will change with 37% left to go. So in one time constant the voltage will have gone most of the way.

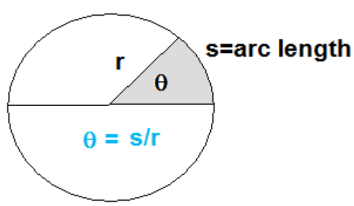
Chapter 6: Trig Equations

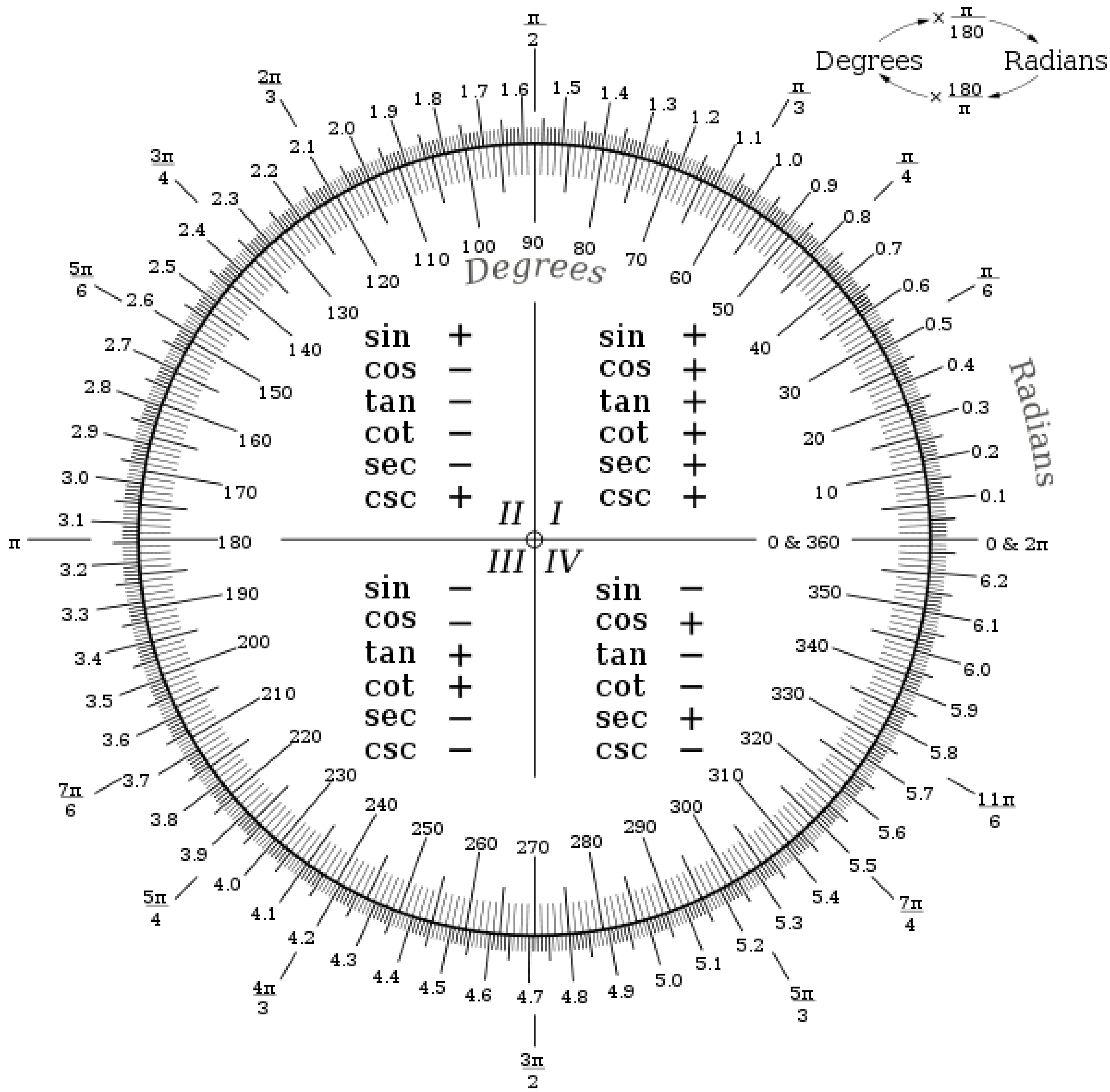
Angle Measures

There are two methods commonly used for measuring an angle. All angle measures are based on the angle being a fraction of a full rotation about a central point. The number or metric we assign to that full rotation sets the scale for the measure of the angles. The most common is the degree. A full rotation is considered 360 degrees. Why 360? There are two major schools of thought on that. First, the old world calendar had 360 days in it, and one degree was the move of the stars in one day. Our precision changed but the definition of a degree remained the same. The second possible source is based on the circle. If a chord is drawn on a circle equal to the radius, the angle thus made was used as the basic unit. This angle was divided into 60 parts. Sixty is the base of the sexagesimal number system. The ancient Sumerians (3^{rd} Millennium BC) used this system. Since six of these chords fit completely into the circle, a complete circle or one rotation would then have 360 parts or degrees.
[source: http://en.wikipedia.org/wiki/Degree_(angle)]



The second measure is again based on the circle. For this method the distance around the arc is divided by the radius. So the angle is a ratio of two length and hence strictly unit less. However, it is common practice to call this measure a radian or rad in reference to the radius of the circle. Since the Circumference of the circle is C=2 π R, the full circle is 2π rad. This leads to a common conversion factor from degrees to radians of $\pi/180$. MathCad uses the radian measure for any work it does with angles like in trigonometry. However, it is an easy process to convert from one to the other as needed. A chart for this conversion is also shown below for quick reference.

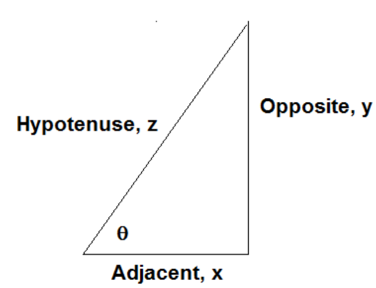




[source: http://en.wikipedia.org/wiki/File:Degree-Radian_Conversion.svg]

Trigonometry Operations

There are three main operations we do with trigonometry. They all relate to the right or 90^o triangle. The term right probably comes from early builders who in erecting building needed to have stones cut at 90^o angles to have their walls stand vertical. Hence only that angle was acceptable or “right”.



The angle Θ is called the base angle and is opposite the 90^o at the base of the triangle.

Three ratios are possible with the three sides of the triangle.

Operation Name	Ratio of sides	Standard notation
Sine	y/z	sin(Θ)
Cosine	x/z	cos(Θ)
Tangent	y/x	tan(Θ)

There are three other ratios that represent the inverse of these three, but for most scientific and engineering calculations these are the only one commonly used. MathCad, of course, understands them all.

Example: In electronics a common alternating voltage is the sine wave voltage. It is expressed mathematically as

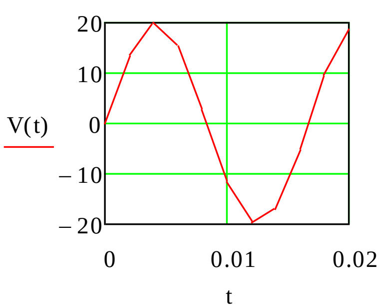
V=Vosin (Θ) where Θ = 2πf t with f as the frequency of the alternating wave.

In MathCad this is …

$Vo := 20 \quad f := 60$

$t := 0, .002 .. .02$

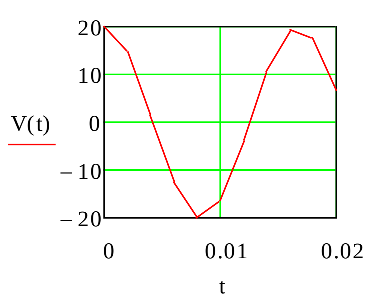
$V(t) := Vo \cdot \sin(2\pi \ f \cdot t)$



V(t) =	t =
0	0
13.691	$2 \cdot 10^{-3}$
19.961	$4 \cdot 10^{-3}$
15.41	$6 \cdot 10^{-3}$
2.507	$8 \cdot 10^{-3}$
-11.756	0.01
-19.646	0.012
-16.887	0.014
-4.974	0.016
9.635	0.018
19.021	0.02

We will see how to do graphs in the next section. This is the common Sine Wave shape seen in electronics. The 20 represents the "peak" voltage of the output. The range of t covers one full cycle and about another ¼ cycle. The cos() operation would produce the following with the same set of parameters.

$V(t) := Vo \cdot \cos(2\pi \ f \cdot t)$



Graphing Trigonometric Functions

In Engineering Technology it is important to be able to work with Sine, Cosine, and Tangent functions. These are all available within MathCAD. One problem however is that the functions use the radian measure to compute the value associated with an angle. In electronics the angles are usually listed in radians so this would pose no problems. However, in other fields and in math classes it is more common to use degrees. MathCAD has a built in feature to convert degrees into radians. Whenever degrees are used, the 'deg' function is multiplied after it. For example:

$\theta := 30\cdot deg$ $\qquad \theta := 0.524$ Radians

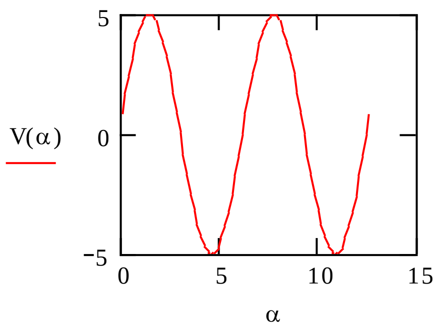
$\beta := 180\cdot deg$ $\qquad \beta = 3.142$ Radians (half way around is equal to Pi)

To demonstrate the graphing of a trig function look at the AC voltage given by the following equation:

$\alpha := 0\cdot deg, 10\cdot deg..720\cdot deg$

$V(\alpha) := 5\cdot \sin(\alpha + 10\cdot deg)$

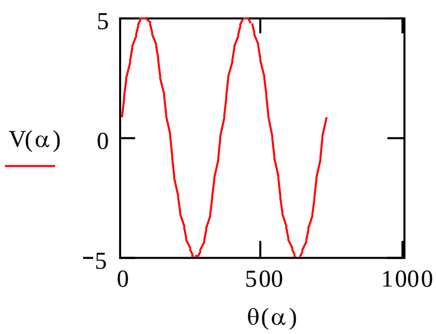
Note: amplitude is 5 and the phase angle is 10 degrees.



Note: the 'α' value plotted in radian measure

In electronics it is often necessary to plot the voltage versus degree or rotation. For this to occur in MathCAD, we must define an angular function. Then we plot this function on our horizontal axis. This will override the default of MathCAD to plot in radians. For example:

$\theta(\alpha) := \alpha\cdot\frac{180}{\pi}$



Note: See how the Theta function allows plotting to 720 degrees. Mathematically these are referred to as parametric equations.

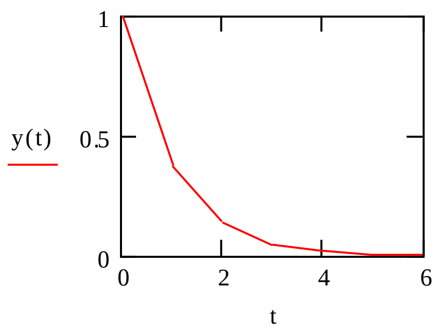
The Exponential Function

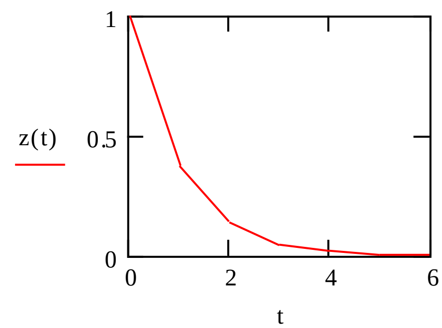
In Chapter 5 the e function was studied. It describes how the charge changes in a capacitor, or how the temperature changes in a cooling slab of metal. There are two ways to deal with this function. First, the function exp(x) may be used. This expression may be familiar from spreadsheets. It is also possible to use the ‘e’ with an exponent.

$t := 0, 1 .. 6$

$y(t) := e^{-t}$

$z(t) := \exp(-t)$





The two graphs look the same because they are functionally the same. Also realize this means that the letter ‘e’ has a special meaning in MathCAD and you should not use it for any other reason.

Parametric Equations

In some situations it is desirable to combine several equations to get the final result. Damped harmonic motion is a good example. We see this type of motion in the bounce of a car, the dying out of a swing’s amplitude, and the decrease of the loudness of the tone of a tuning fork. The equations are:

$t := 0, 1 .. 40$

The sine wave vibration

$V(t) := 5 \cdot \sin(t)$

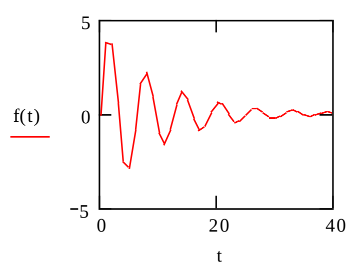
The damping term

$y(t) := e^{-.1 \cdot t}$

These are called parametric equations.

The equation of the combined motion

$f(t) := y(t) \cdot V(t)$

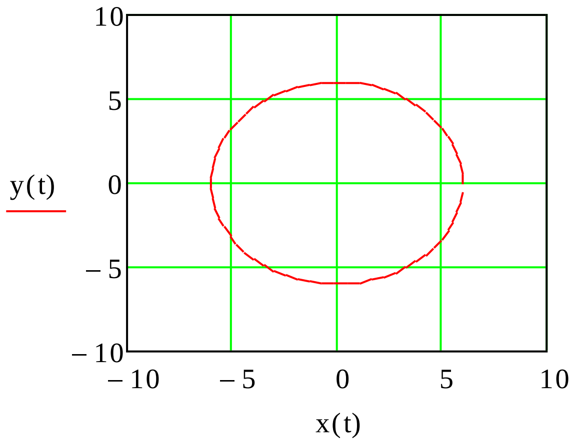


Another example of these combined equations would be the equation of a circle.
In parametric form this is
X=cos (Θ) and y=sin (Θ)
MathCad allows the following example,

$$t := 0, 0.1 .. 2\pi$$

$$x(t) := 6\cdot\cos(t)$$

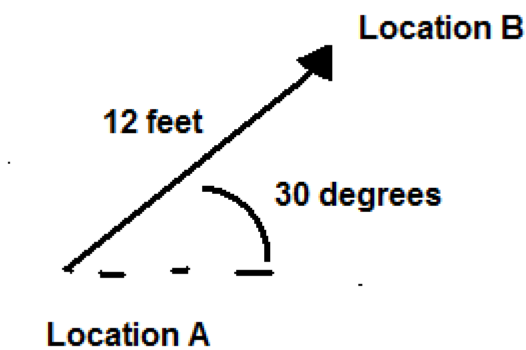
$$y(t) := 6\sin(t)$$



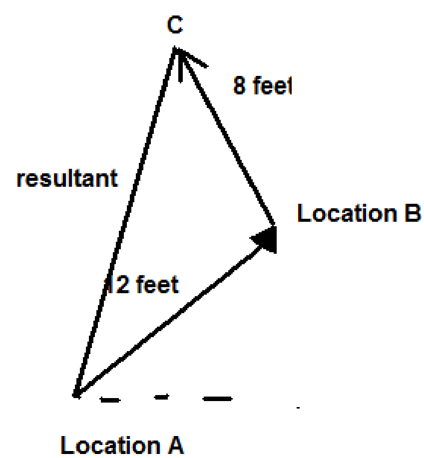
The radius of the circle is 6. See how that is the limit of calculation. The last value of t was slightly small that the full circle of 2π, note the circle does not quite close.

Chapter 7: Vectors and Phasors

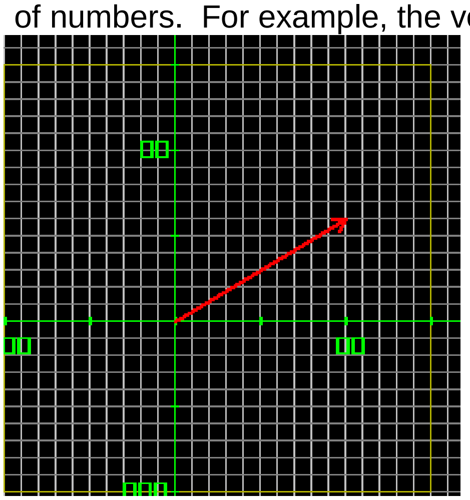
One concept important to both mechanical and civil technology is that of movement. We characterize objects moving about by concepts of location and speed. First let's explore location. When we are at a particular location we could define our location in a variety of ways. We might say we are at 1329 Main Street on the second floor in room 205. If we had a GPS system, it would give us coordinates in terms of latitude and longitude to within a few inches of our location. Really good systems can also provide elevation above or below sea level. If you moved, we could locate you by the amount you moved from your present location. This is called the displacement.



Here the displacement is 12 feet at an angle of 30 degree from the original position A. The Line is called a **vector.** A vector has magnitude (12 feet here) and direction (30 degrees). If you move again, we would represent that move with a second vector added to the end of the first. As shown below.



The new location is 8 feet in a new direction. You could get to the new location by moving along a different path called the resultant. This new vector represents the effect of the action taken.
The best way to represent vectors is as an x,y pair of numbers. For example, the vector from A to B



would be
X= 10.4 and y=6
We get this from
the equations below:

$y = V \sin(\Theta)$ and $x = V \cos(\Theta)$ Where

Θ is the angle with the horizontal and V is the length of the vector. For the vector above

$Y = 12 \sin(30)$ and $x = 12 \cos(30)$

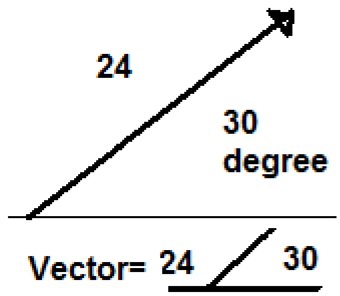
These can be calculated with your TI83 in short order. But, we want to use MathCad.

Mathcad and Vectors or Phasors

In Physics, Mechanical, and Civil Engineering Technology we deal with vectors in many forms. In Electronic Engineering Technology when dealing with AC circuits with reactive elements we deal with a similar math we call phasors, which describe The phase angle and magnitude relationship. This document will describe how to use MathCad to work with these quantities.

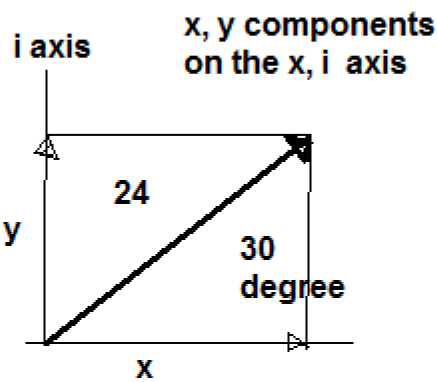
First a vector has both magnitude and direction. For phasors we often represent these quantities as value /_ degree. In electronics the value is either impedance, voltage or current, and the degree is the phase angle. As a vector this would be magnitude and direction. So for example a phasor of 24 V at a phase angle of 30^o would be

$$V1 = 24/\_\ 30^o$$



To use this with MathCad we must reduce it to an x,y pair. The x,y pairs are also called components of the vector. To use MathCad for this type of number it must be placed in the form

$$V1 = x + yi$$



where **i** is the square root of -1 and this is called a complex number with a *real* and *imaginary* part. To get at the x,y components

x= Value cos (Θ) and y= Value sin (Θ)

When the vector or phasor is in the a+bi form they can be added, subtracted, multiplied, and divided as any number can be. Then with a little work we can have the answers in any number display format we wish.

When the number is in the a+bi format the value **a** is called the real part and the **b** is called the imaginary part. There nothing *unreal* about the value b. The use of the word imaginary is unfortunate in that sense. It is just stating that the value is tagged with Sqrt(-1)= i. The i quantity is on the calculator toolbar.

MathCad will handle numbers in this format as any other number. It also has special functions to convert this format to value, angle format (often called polar form.).

To convert polar format to a+bi or complex format the MathCad process is

$Z4 := z\cdot\cos(\Theta) + z\cdot\sin(\Theta)\cdot i$ Where z is the value in polar

To convert a+bi to polar form use

$Vectormag := \sqrt{(Re(Z3))^2 + (Im(Z3))^2}$ $Vectorangle := arg(Z3)\cdot\frac{180}{\pi}$ Here the Re() function represents the real or x value of the complex number and Im() is the y part of the complex number. The function arg() returns the angle formed by x and y. These functions are case sensitive.

The 180/π is to convert from radians to degrees as MathCad always uses radian measure.

Sample Vector Problem

$r := 45$ feet $\Theta := 38$ degrees

Find the x,y pair

Convert to radians: $\Theta 1 := \Theta \cdot \frac{\pi}{180}$ A second vector

$x := r \cdot \cos(\Theta 1)$ $y := r \cdot \sin(\Theta 1)$

$x = 35.46$ $y = 27.705$

$Vector := x + i \cdot y$

$r2 := 30 + 14 \cdot i$

$NewVector := Vector + r2$

$NewVector = 65.46 + 41.705i$

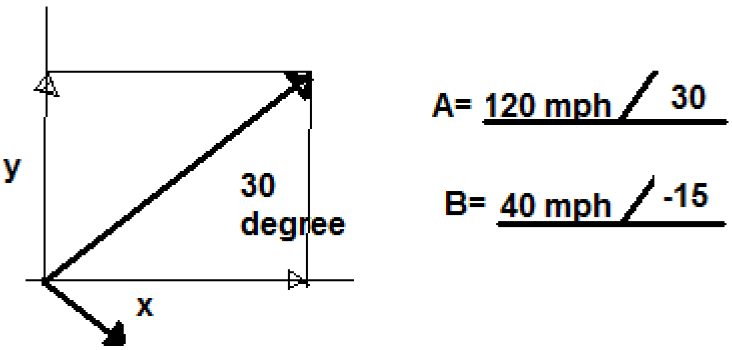
$$Vectormag := \sqrt{(Re(NewVector))^2 + (Im(NewVector))^2}$$

$$Vectorangle := arg(NewVector) \cdot \frac{180}{\pi}$$

$Vectormag = 77.617$ value

$Vectorangle = 32.501$ degrees

In this example use was made of each item discussed above.
Notice also you can use any thing for the variable name even complete words –just no spaces! Here we added only two vectors but any number desired can be added or subtracted as needed. For example in electronics we may have a resistance phasor, a capacitive phasor and an inductive phasor or any combination to be added together. Suppose we have a plane traveling at 120 mph with a cross wind of 40 mph as shown below:



A= 103.9 +60 i   B= 38.6 – 10.4 i
Using the conversion from polar to complex format.
To “resolve these vectors add them together. Z=A +B

$Z := A + B$

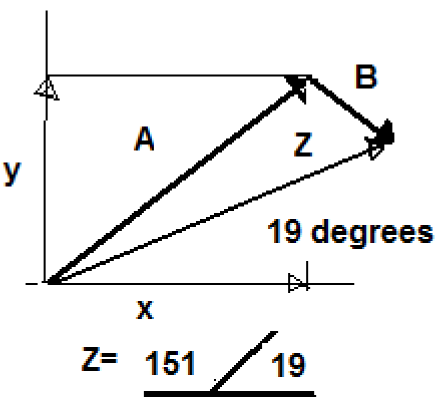
$Z = 142.5 + 49.6i$

$\text{Vectormag} := \sqrt{(\text{Re}(Z))^2 + (\text{Im}(Z))^2}$

$\text{Vectorangle} := \arg(Z)\cdot\frac{180}{\pi}$

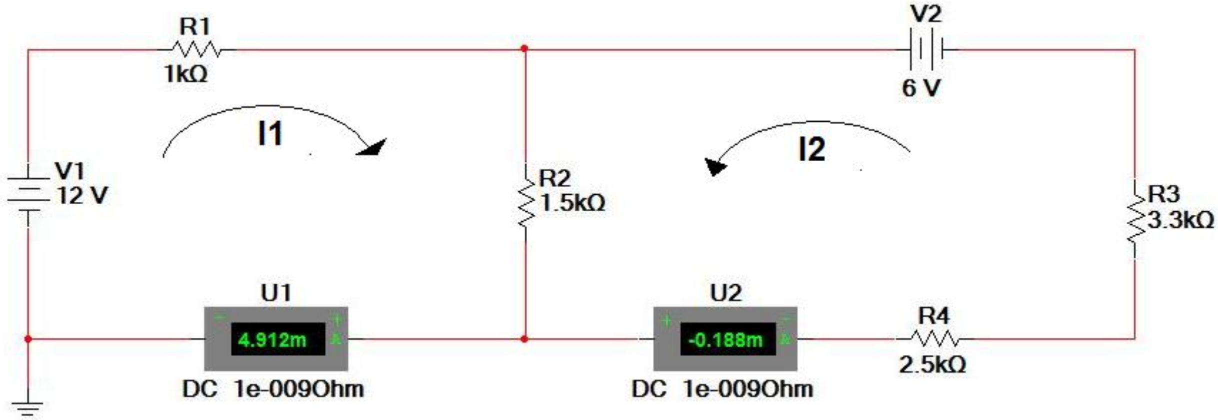
$\text{Vectormag} = 150.885$

$\text{Vectorangle} = 19.191$



Chapter 8: Simultaneous Equations

In Electronics it is common to need to solve multiple equations simultaneously - in circuit analysis or resistor networks or in transistor bias situations. The circuit below is typical of the type encountered in electronics.
Here we need to write the equations based on Kirchoff's Voltage Law.



We have assumed two currents as shown from + to – for each battery source. Bsed on this convention then the voltage drop on R1 is –I1 R1, and on R2 it is –(I1+I2) R2. The negative is because for the current to move as shown the right side of R1 would be negative with respect to the left side. The equations then become

-(I1 + I2) R2 + V1 –i1 R1 =0
-(I1 + I2) R2 - I2 R4 – I2 R3 +V2 =0

Now we could solve these by a variety of methods, substitution of one equation into another, matrix/determinate techniques etc. However, let's use MathCad and let the machine do the heavy lifting.

We start this process by setting up a construct called a Given..Find sequence. We need to tell MathCad what we know about the situation. This is done by providing the known values, the equations (remember these are like sentences of explanation to MathCad), and a "guess" and the right answers.

A few precautions here, first set the units to **None** so we don't get sidetracked with conversions. We will deal with units in a separate section of this book.
The solution will appear as shown below. The "text comments" are in blue, the math is in black.

Solving Simultaneous Equations

Provide known quantities as local variables

$R1 := 1 \qquad R2 := 1.5 \qquad R3 := 3.3 \qquad R4 := 2.5$

Guess answers

$V1 := 12$

$V2 := 6$

$I1 := 1 \qquad I2 := 1$

Given

$$-I1 \cdot R1 - (I1 + I2) \cdot R2 + V1 = 0$$

$$-(I1 + I2) \cdot R2 - I2 \cdot R4 - I2 \cdot R3 + V2 = 0$$

$$\mathrm{Find}(I1, I2) = \begin{pmatrix} 4.912 \\ -0.188 \end{pmatrix}$$

Note, use Boolean equals for the two equations, and the solve equals for the Find. The units here are V is in volts, the current I will be in milliamperes, and the resistance is in kilohms. These are the units common for electronics, but MathCad would not use this scheme normally, hence turning off units. From this the currents are
I1 = 4.912 mA and I2 = -0.188 mA.

The negative is because we selected the “wrong” direction for the I2 current. No biggy, since the math will tell us we went the wrong direction. Note this agrees with the values Multisim predicts.

The number of equations involved is not critical. We can as easily do 5, 8, 10 equations as the two above. Plus, if we want to “play with the values to arrange a series of “what ifs”, this is easily done. For example if we change the battery form 6 to 10 volts what happens?

Solving Simultaneous Equations

Provide known quantities as local variables

$R1 := 1$ $R2 := 1.5$ $R3 := 3.3$ $R4 := 2.5$

Guess answers

$V1 := 12$

$V2 := 10$

$I1 := 1$ $I2 := 1$

Given

$$-I1 \cdot R1 - (I1 + I2) \cdot R2 + V1 = 0$$

$$-(I1 + I2) \cdot R2 - I2 \cdot R4 - I2 \cdot R3 + V2 = 0$$

$$\mathrm{Find}(I1, I2) = \begin{pmatrix} 4.538 \\ 0.437 \end{pmatrix}$$

Note the current changes values, plus the I2 is now in the correct direction coming out positive. Of course once the structure is set up as above, we can change or modify the equations or add equations. This will serve as a template for solving the simultaneous equations.

Chapter 9: Statistics

Dialog: Basic Statistics

In industry it is important that we make products of high and equal quality. It is not good for our business if only one of 10 TV sets will last through the 3 year warranty. We would hope they all will. Every set that fails is lost profit and ultimately lost jobs and business. Statistics is used by the quality control department to gage how well the manufacturing process is working. Today with process like ISO 9000 and 6 Sigma being used in quality control some knowledge of the basic statistical functions is important even for the technician.

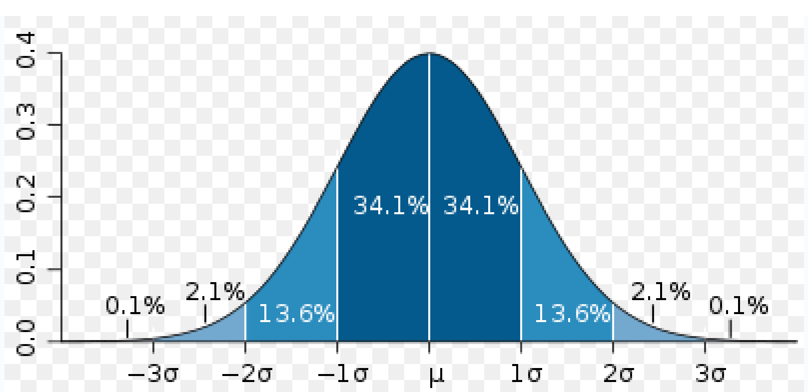
In many situations multiple values are taken for a particular parameter. They are averaged to get a representative number. We do this by adding all the values together and dividing by the number of values.

Now this would not be necessary if we always got the same number from a process, but this will not happen. There are many factors involved in any operation and a small change in any one can change the result. if we have no control or way of predicting their size, the errors are called random. If the errors are not random, it may be possible to eliminate them with better control. When we have random errors, the mathematics of statistics will provide a prediction of the error involved. This is done through a formula shown below.

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

Here N is the number of items and μ is the average value. The x_i is each of the values measured. The σ (pronounced sigma) is the standard deviation. Although this is an imposing equation we never really have to deal with it, the calculations are built into MathCad.

Now what does all this mean? For randomly produced error, the values will fall about the average (usually called the mean) in the following ways:



(from Wikipedia, http://en.wikipedia.org/wiki/File:Standard_deviation_diagram.svg)

For each σ the percentages shown will be the number of values of the sample in the given range. So for example, 68.2 % will fall within + or - one σ. It is common to express an answer as

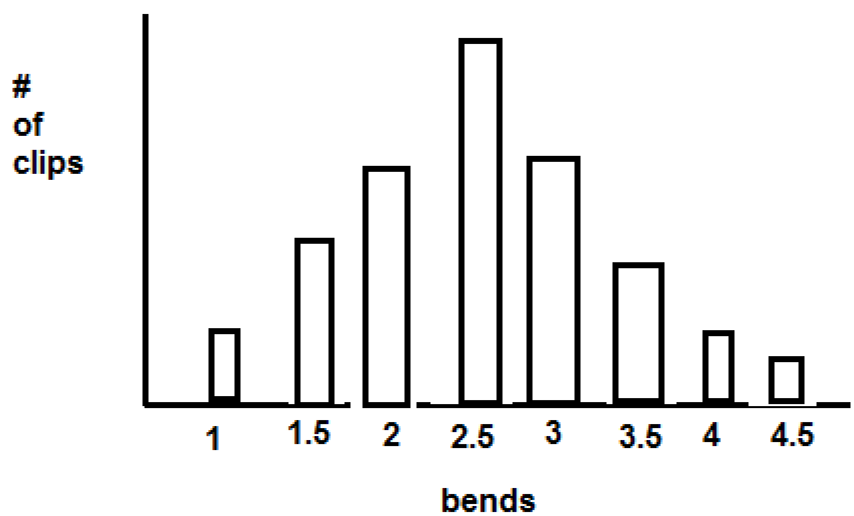
36.8 +/- 2.3 mean +/- σ

This will tell the reader how your value *bunched*. The relative width of the curve tells us then how tight our measurements are. If the σ is large there can be a wide range of possible values if small the range will be much smaller. Nearly all the values should be within +/- 3σ.

Histogram

The graph above is called a histogram. It represents in graphic mode the way the data is distributed. (Sometimes this is called a distribution curve or chart). To produce one you first divide your data into value groups. Say all values of 1, all of 1.5, 2.0, 2.5, etc. then graph them as a bar graph. It might look something like the figure below. The centers of the bars should follow the distribution curve above.

Histogram of Samples



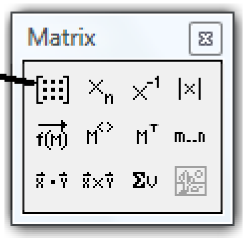
To calculate these quantities with MathCad it is necessary to introduce a new type of local variable. This is needed to input a long series of non-interval numbers – the raw data.

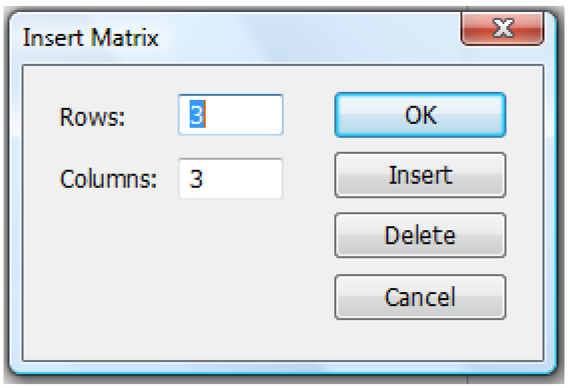
Enter a variable as you usually would for example

Xdata :=

Then click on the matrix toolbar upper left corner. This will bring up the Insert Matrix box below. For the number of rows this should be the number of data points to be entered. The Columns should be one.

click here





This will produce a very long column for data entry >>>

$$xdata := \begin{pmatrix} 2 \\ 3.3 \\ 4.2 \\ 5.1 \\ 6.9 \\ 1.0 \\ 2.4 \\ 3.3 \\ 4.5 \\ 5.0 \\ 7 \\ 3.4 \\ 2.5 \\ 2.2 \\ 1.8 \\ 3.3 \\ 4.3 \\ 4.2 \\ 3.4 \\ 3.9 \\ 2.9 \\ 2.8 \\ 3.1 \\ 3.2 \\ 2.9 \end{pmatrix}$$

This column will represent the data to be treated.
To find the Mean we will use a MathCad function.

Here the value of average is gotten using the mean() function.
Remember functions are **case sensitive**.

$$average := mean(xdata)$$

$$average = 3.544$$

Now to get the standard deviation we use the Stdev(). Note case!

$$dev := Stdev(xdata)$$

$$dev = 1.418$$

Therefore our data is 3.544 +/- 1.418

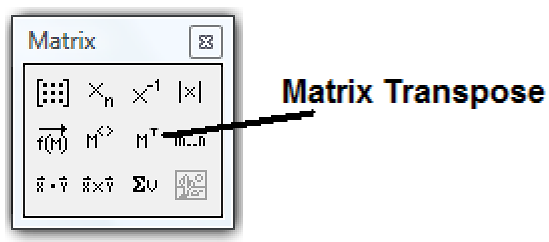
This will handle the majority of the statistics work we might be expected to carry out on a routine basis. However, it would be nice to offer a histogram of the data as well. This was a lot of work by hand. With MathCad it takes a little setup but the results are very easy to produce by comparison to the hand work. And remember, Math Cad doesn't care if there are 10 or 10,000 data points to deal with.

For a histogram we must first determine the number of bins of groupings we are going to use. Here the intended purpose of the histogram is important. If you provide too much detail the graph will be too cluttered to be useful. A good number is say 10 bins or less.
For this exercise we will use 10 bins. The variable N will represent the bins.

$N := 10 \qquad i := 0..N$

$\text{interval}_i := 1.4 + \frac{i}{N} \cdot 4$

The variable **i** is to represent the element size in our new array of values. The trick here is to select values that will match the data. I have chosen the first bin to be 1.4 the size of the standard deviation. Then I have added 1/n th, 2/n th, etc. to this for each bin generating the other bin values. The number 4 is larger than any value in the data set I expect (a total of 4+1.4 or 5.4) What you are doing is defining another array or matrix called interval. Now to see the bin values you have set up use the MathCad transpose function. Type interval then the m^T and regular = key.



$\text{interval}^T =$

	0	1	2	3	4	5	6	7	8	9
0	1.4	1.8	2.2	2.6	3	3.4	3.8	4.2	4.6	...

This shows each of the bins that will be populated.

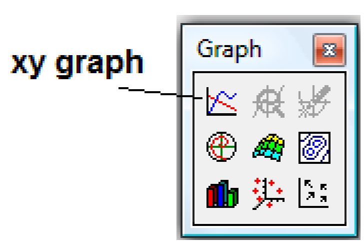
Find how many of each data point falls into each bin we define a new variable called weight as below.

$\text{weight} := \text{hist}(\text{interval}, \text{xdata})$

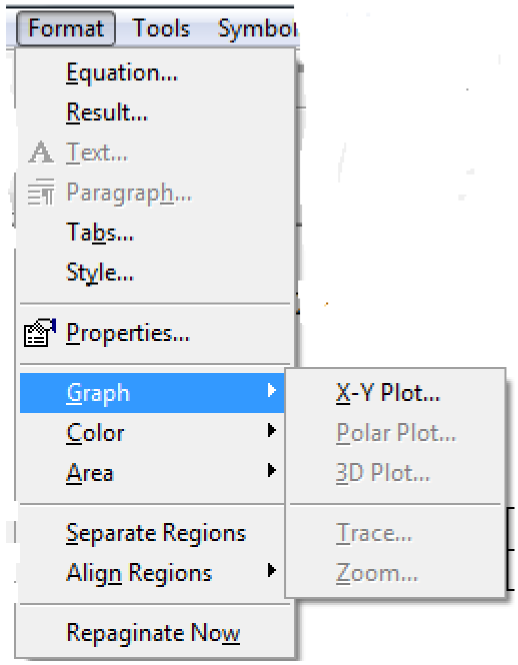
$\text{weight}^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	2	3	3	5	2	1	4	0	2

So we see that in bin number 4 which goes from 2.6 to 3.4 we have 5 values in our data set. We could now draw our histogram. But wait! This is MathCad won't it do the drawing for us? Yes! Yes! Yes! From the graph toolbar select xy graph.

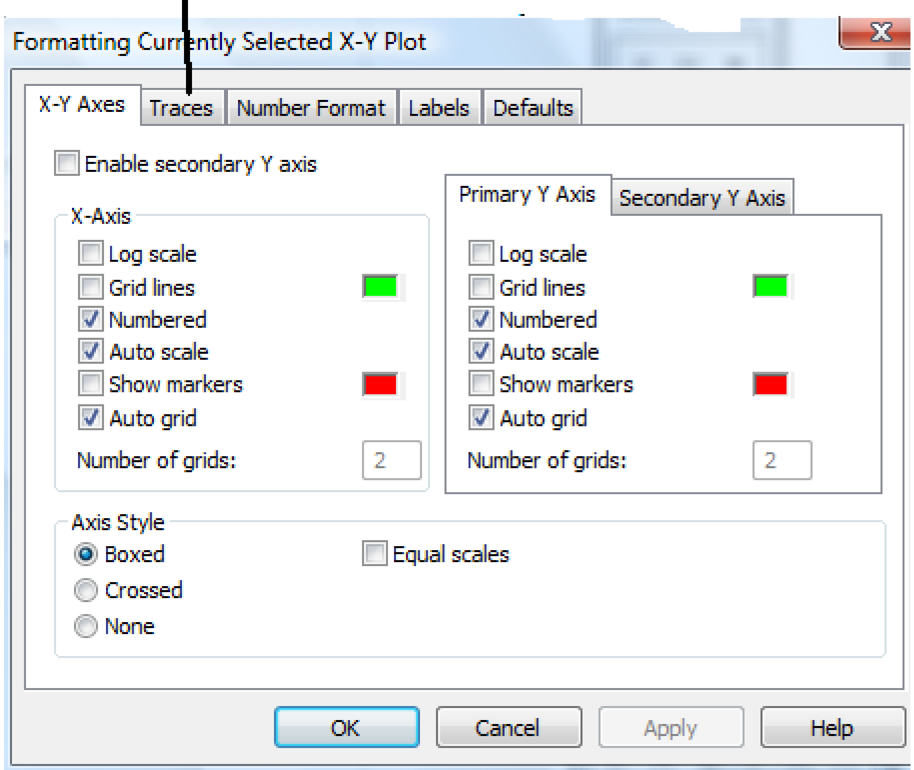


Then format xy graph,

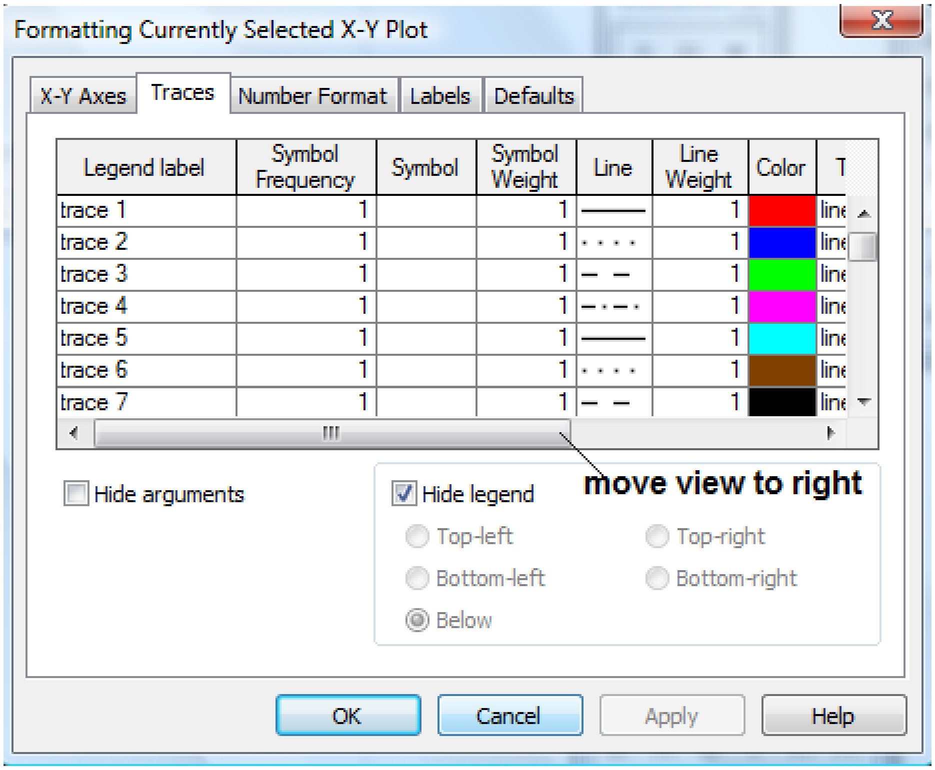


This will bring up two more screens. First this one

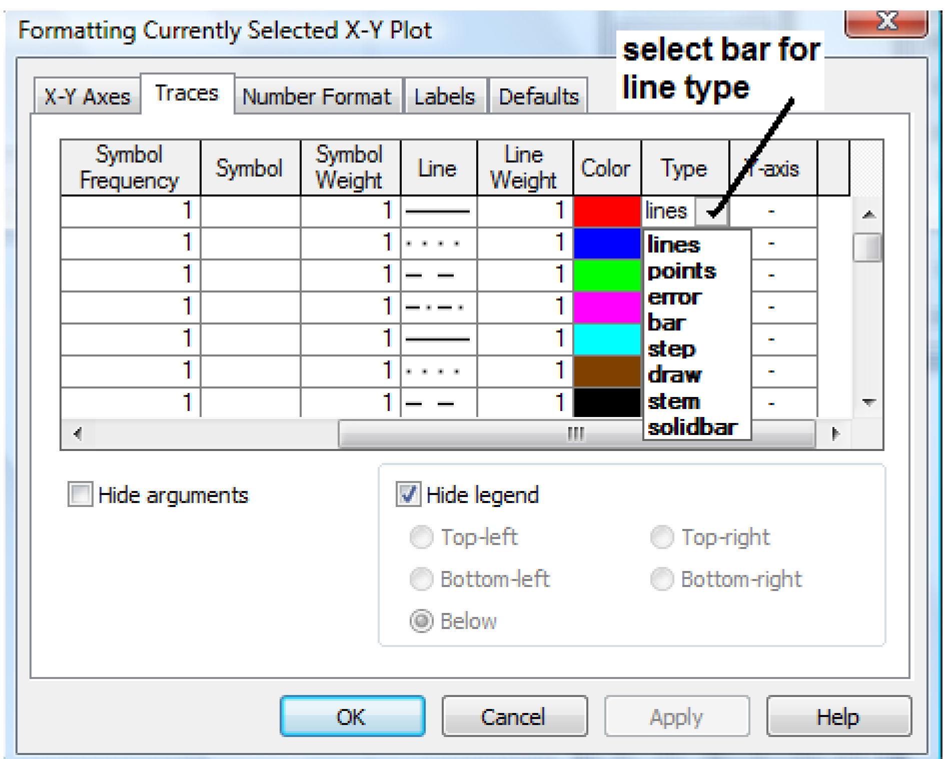
select traces



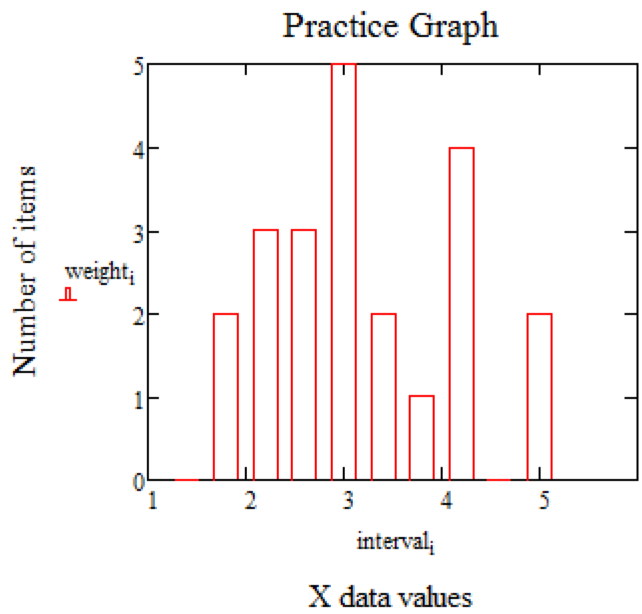
Then



Followed by…



Now you have set up your graph as a bar graph in xy format. We want to plot interval on the x axis and weight on the y axis. With a little use of the formatting box we can get the following.



A very nice histogram, that can be copied and pasted into reports, etc.

Once one example is set in MathCad, you can use the problem as a template to solce any number of similar statistics situations. That of course is the further power of MathCad, do it right once, and repeat it again and again.

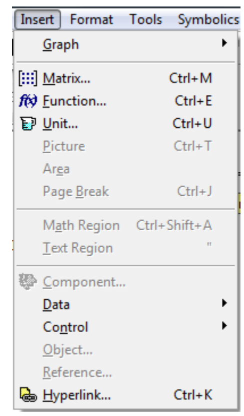
Chapter 10: Units

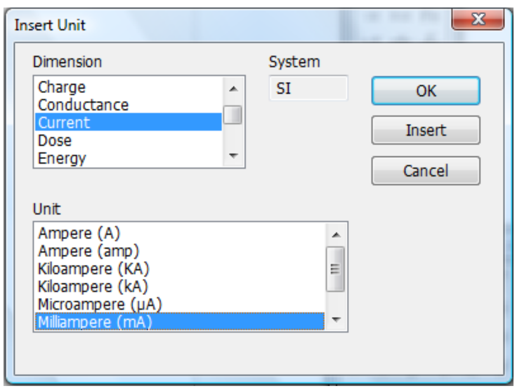
In MathCad many unit systems are built into the operations. The default system is SI, for Scientific International. In this system current for example is defined in Amperes fractions of Amperes. Resistance is measured in Ohms. The example below will demonstrate how that can be used.

Find the voltage on a resistor of 2,500 Ohms with a current of 3 mA.

First enter the variables:

$I := 3$ Select from the Insert tab the Unit (Ctrl-U) and we will get the additional menu below. From this pick current as mA.





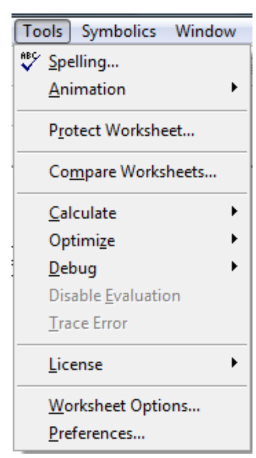
Repeat the process for resistance and select KΩ. The results will automatically appear in Volts, V.

$I := 3mA \qquad R := 2.5K\Omega$

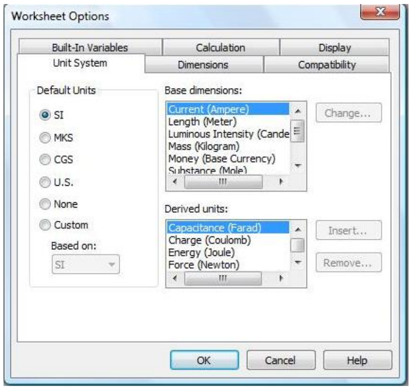
$V := I \cdot R$

$V = 7.5V$

On some occasions we do not want to use units in the calculation. To signify no units we select the following…



Click on Worksheets and then select None for the systems of units and press OK.



This will not turn off all unit definitions. MathCad will still have some basic definitions like length, mass etc define but in general terms. If a problem comes up on using a variable name etc, check that it is not a predefined quantity. If so pick a slightly different name for the variable. This mostly a problem when single letter variable are used.

The units feature will force the calculation to agree with the standard units. For example, lets' look at a calculation of heat transferred in a calorimetry experiment. Suppose a block of aluminum has 200 grams of mass and is heated to change its temperature by 120 ^{o}C. The specific heat of aluminum is shown below from a set of tables.

$$m := 200\,\mathrm{gm}$$

$$c := .22\,\frac{\mathrm{cal}}{\mathrm{gm}\cdot\Delta{}^{\circ}\mathrm{C}}$$

$$\Delta T := 120\,\Delta{}^{\circ}\mathrm{C}$$

$$Q := m\cdot c\cdot\Delta T$$

$$Q = 2.211\times10^{4}\,\mathrm{J}$$

The results are in Joules even though we started with calories. This is because Joule is the standard energy unit for SI units. Divide the result by the conversion of 1 calorie = 4.18 Joules.

Q= (22,110 J)/4.18 = 1,263 calorie

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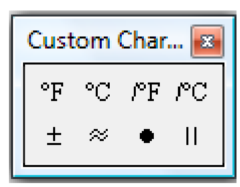
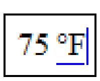
There are other toolbars useful with units. Select

View > toolbars > custom Resulting in

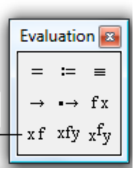


The interesting feature of the temperature characters is it does conversions for you.

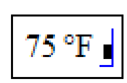
First type the temperature and add the degree unit form the custom menu above. Then press the xf operation from the evaluation menu bar.



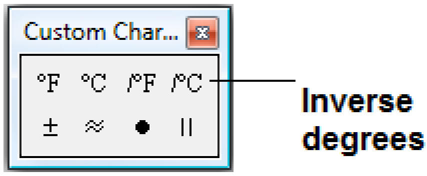
xf operation



This will result in the display as at right.



Next select Inverse degree operation from the custom menu toolbar. The result should match below.



$75\ °F\ /°C$

The last step is to use the evaluate = to get the answer in degrees C.

$$75\ °F\ /°C = 23.889$$

This may seem like a lot of steps, but it is quicker than writing the full equations for the conversions.